A Model of Cryptocurrencies*

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Abstract

This paper develops a model to examine decentralization of online platforms through tokenization as an innovation to resolve the conflict between platforms and users. By delegating control to a collection of preprogrammed smart contracts, tokenization creates commitment devices that prevent a platform from abusing its users. This commitment comes at the cost of not having an owner with an equity stake who, in conventional platforms, would subsidize user participation to maximize the platform's network effect. This trade-off makes utility tokens a more appealing funding scheme than equity for platforms with weak fundamentals. Our analysis further highlights that token prices are determined by the marginal user's convenience yield, in contrast to equity, whose payoff is determined by the average user.

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The proliferation of the digital economy and the recent rise of the fintech industry have led to two important trends. The first is that a sizable number of digital platforms have funded their development and operations through the issuance of cryptocurrencies or tokens. According to Allen, Gu and Jagtiani (2020), for instance, as of May 2020 there exist 4,136 cryptocurrencies, not including many that have failed. While rampant speculation and volatility are often observed in this asset class, its growing popularity raises important conceptual questions about the benefits and costs associated with the tokenization process and the determinants of token prices. The second trend is the growing tension between digital platforms and their users as online platforms, such as Amazon, Google, and Facebook, become pervasive in our everyday lives. Their large networks of users not only facilitate monopoly power in pricing but also extensive access to users' private data for advertisement targeting and sale to third-party vendors.[1] These privileges are subject to abuse, as reflected by ongoing antitrust investigations into big-tech companies and the enactment of data privacy regulations in the European Union, the United States, and Japan. Such conflict between online platforms and their users represents a unique challenge to their design and raises questions about whether they could be disintermediated to protect consumers.

In this paper, we link these trends by arguing that tokenization represents an innovative effort to resolve the tension between platforms and users, similar to how corporate finance has developed governance tools to manage the classic tension between firm managers, who control the firm's operations, and firm owners, who own the firm's assets. Indeed, the success of Bitcoin, the first cryptocurrency to achieve unprecedented popularity across the world, was largely built upon the notion that delegating the issuance of the cryptocurrency to precoded computer algorithms would free its users from potential abuses by central bankers, who control the supply of traditional fiat currencies and may increase the supply for their own interest but at the expense of current holders of the currency. The success of Bitcoin has further stimulated strong interests in using cryptographical technologies to design decentralized platforms that delegate issues of governance and enforcement to users through a collection of preprogrammed smart contracts, thereby preventing platform owners from abusing their users. Harvey et al. (2020), for instance, provide a roadmap for how crypto-based technologies can decentralize various aspects of the financial industry.

[1] There is extensive literature exploring how online platforms' extensive access to user data may allow them to price discriminate users, e.g., Taylor (2004), and take advantage of users' personal vulnerabilities such as weak self-control, e.g., Liu, Sockin and Xiong (2020).

Decentralization through tokenization divests initial equity holders of their ownership and control of a platform. Allocating the control to users makes it possible to prevent user abuse. This benefit, however, comes at the expense of removing any owner who would subsidize user participation to maximize the platform's network effect. As network effects are essential for the success of online platforms, conventional platforms typically devote substantial resources to subsidize user participation to amass a large user base. For example, Google and Facebook offer free search and social networking services to attract users. The equity holders of these platforms bear the initial costs of subsidizing user participation to maximize future advertising revenue, which increases with the size of the user base.

In this paper, we develop a model to investigate the trade-off induced by decentralization between safeguarding users and subsidizing their participation in the presence of network effects. We also use this framework to address the determinants and properties of utility token prices, which represent the most common form of tokenization in practice.

Our baseline model features an online platform that facilitates bilateral transactions among a pool of users. There are three dates. At time 0, the developer of the platform chooses to fund the platform by issuing either conventional equity or a cryptocurrency. The choice of the funding scheme also determines the control and ownership of the platform in the subsequent periods. At time 1, potential users choose whether to join the platform, subject to a personal cost of downloading the necessary software and becoming familiar with the platform's rules and user interface. After joining the platform, a user can benefit from matching with other users to make bilateral transactions at both times 1 and 2. We model a user's transaction need by his endowment in a consumption good and his preference of consuming his own good together with the goods of other users. As a result of this preference, users need to trade goods with each other, which can occur only on the platform. Consequently, there is a key network effect—each user's desire to join the platform grows with the number of other users on the platform and the size of their goods endowments.

To provide a sharp characterization of the key conceptual issues, we focus on two archetypal funding schemes for the platform. One is the conventional equity-based scheme in which equity conveys both control and (residual) cash flow rights. If the developer issues equity, it leads to a group of equity holders that is represented by an owner who takes ownership and control of the platform. The owner would choose to provide a subsidy at time 1 to attract the marginal user, whose own transaction need is relatively low and who is otherwise not incentivized to participate on the platform without the subsidy. The participation of the marginal user, however, makes it easier for other users to find transaction partners and consequently maximizes the network effect. As the owner can profit from charging transaction fees that increase with the transaction surplus on the platform, he would internalize the participation cost of the marginal user by providing a subsidy to all users. Control of the platform, however, also allows the owner to abuse users at time 2, after the platform collects extensive user data at time 1.

We consider a particular form of user abuse—the owner may choose a subversive action (such as pursuing aggressive advertising strategies or selling user data to third parties, as is sometimes observed in practice), which benefits the owner at the expense of all users. Intuitively, the owner would choose this action only when the transaction fees from the platform fall below the gains from abusing its users. Interestingly, while choosing this subversive action may benefit the owner ex post at time 2, the owner is strictly better off ex ante at time 1 if he can precommit to not taking such an action, because anticipation of the owner's taking the subversive action discourages potential users from joining the platform initially, and this abandonment is magnified by the network effect. It is impossible to precommit under the equity-based scheme, as the owner can always choose to reverse any previous commitment at time 2. This demand for precommitment motivates tokenization.

Alternatively, the developer may adopt a token-based scheme by issuing utility tokens, which are widely used in practice and considered the canonical crypto-based tool for decentralization. While utility tokens also confer voting rights to holders, similar to equity, they are a claim to the platform's services and not to its (residual) cash flows. Under this token-based scheme, the owner sells tokens to users to participate on the platform instead of charging fees. By cashing out from issuing tokens to users who join the platform at time 1, the developer leaves control of the platform at times 1 and 2 to users through precoded algorithms, which can conveniently foster a precommitment not to abusing users by requiring their consent. Although users, as the holders of the tokens, can vote on changes to the platform and these algorithms, they would not agree to adopt any action that would hurt themselves. This captures the key appeal of tokenization—giving ultimate control of the platform to users through decentralization. This benefit, however, comes at the cost of not having an owner with an equity stake who would subsidize user participation to maximize the platform's network effect.

A comparison of these two schemes leads to several key insights: First, the token-based scheme with utility tokens is more appealing for platforms with relatively weak demand fundamentals (i.e., aggregate transaction needs by users). Under the equity-based scheme, users' concerns about the owner subverting the platform are particularly high when transaction fees to the owner are low, which makes the precommitment created by tokenization particularly valuable. Consistent with this observation, we show that for a given level of concern about user abuse, user participation, developer profit, and social surplus are all higher under the equity-based scheme when the platform fundamental is sufficiently high; for a given level of platform fundamental, in contrast, user participation, developer profit, and social surplus are all higher under the token-based scheme when the concern about user abuse is sufficiently high. This dichotomy leads the developer to choose the token-based scheme when his prior belief about the platform fundamental is relatively low. Such a stark implication is consistent with casual observations of the high failure rates of tokenized platforms and can be systematically tested by future empirical studies. More generally, our analysis suggests that decentralization is desirable on platforms for which the tension between owners and users is sufficiently severe.

We acknowledge that tokenization requires implementing a certain consensus protocol to accomplish the intended decentralization, which is challenging in practice and may introduce conflicts between users and record keepers, who record and validate transactions on blockchains, and even conflicts among record keepers. As such conflicts are not our focus, we take as given the frictionless implementation of a consensus protocol to focus on how the resulting decentralization from tokenization affects user participation and welfare. Although we abstract from these important issues, our analysis highlights a key trade-off that decentralization introduces between empowering users with control of digital platforms and subsidization of their participation. More sophisticated token arrangements may be able to better balance this trade-off than utility tokens. We nevertheless note that this key trade-off can reemerge in new forms under alternative token arrangements,[2] as well as through the implementation of the consensus protocol.[3] As digital decentralization represents a promising

[2]For example, by conveying both control and cash flow rights, equity-like tokens may incentivize some holders who may not be major users of the platform to amass enough tokens to collect rents from effectively becoming owners, which would reintroduce the conflict between owners and users.

[3]See Section 1.4 for a review of the burgeoning literature that analyzes the efficiency and economic consequences of alternative consensus protocols. A particular concern is that record keepers may gain concentrated control of the platform under certain consensus protocols, thereby effectively becoming owners and reintroducing the conflict between owners and users.

intersection of economics and technology, our analysis represents a first step in the optimal design of decentralized platforms.

Second, the model also highlights a sharp distinction between the prices of equity and utility tokens. Under the equity-based scheme, the owner's profit is determined by the aggregate transaction fees collected by the platform, which ties the equity price to the transaction surplus of the average user. In contrast, under the token-based scheme, the token price is determined by the indifference condition of the marginal user, whose transaction surplus from participating on the platform determines the equilibrium token price. This dependence on the marginal user consequently distinguishes token and equity prices as claims to the cash flows of the underlying platform.

To further explore the implications for token pricing, we expand the model to incorporate overlapping generations of users on a tokenized platform. In each period, a generation of users chooses to purchase tokens to participate on the platform and then sell the tokens in the next period to users of the next generation. The resale of tokens allows users' expectations and sentiment about the token's future resale price to directly affect each user's token purchase and the platform's user base. In equilibrium, the marginal user is indifferent between the net cost of participating on the platform, which includes the private cost, the cost of carrying the token for one period, and the convenience yield from the platform. Since the valuation of the marginal user is more sensitive to the platform's network effect than the average user, token prices are more sensitive than that of equity to the size of the user base. This contributes to the vulnerability of the token platform as the expected capital gain feeds back into the decision of the marginal user to join the platform.

Our dynamic model provides a rich set of empirical predictions about token price fluctuations and expected token returns. First, the token price is directly related to the platform's user base because of the network effect. This implication is consistent with the empirical observation that token prices and the size of the user base tend to positively comove (Bhambhwani et al. (2020)). The network effect also makes the user base particularly volatile for platforms with weak fundamentals. Second, our model highlights the convenience yields of users as a key determinant for expected token returns, as opposed to conventional risk premia for equity returns. Specifically, our model implies that expected token returns are higher for weak platforms, which offer lower convenience yields to users. The premise of this implication is consistent with a common empirical finding of the lack of conventional equity market risk premia in cryptocurrency markets (Hu et al. (2018), Liu and Tsyvinski (2019)). To the extent that users' convenience yields from a platform persist over time, expected token returns are also persistent and predictable by factors that can predict these convenience yields. Through the marginal user's platform participation, our model also predicts a role for both news and investor sentiment, possibly driven by fluctuations in the price of Bitcoin, to explain token price fluctuations and expected returns.

Related literature Our paper is related to the growing literature on Initial Coin Offerings (ICOs) and their comparison to traditional financing schemes. Different from our focus on the conflict between platforms and users, many of these studies focus on the classic conflict induced by moral hazard between an entrepreneur and outside investors. Chod and Lyandres (2019) and Chod et al. (2019), for instance, show that utility token financing is preferable to equity in mitigating the underprovision of effort by an entrepreneur, but leads to underinvestment and an underproduction of goods that are sold in advance. Catalini and Gans (2019) and Gan et al. (2020) compare utility tokens to revenue-sharing and equity to profit-sharing, with the former emphasizing that tokens facilitate competition and coordination among buyers and the latter that equity is better in aligning the incentives of entrepreneurs and speculators. Malinova and Park (2018) find that tokens can finance a larger set of ventures in the presence of entrepreneurial moral hazard but are inferior to equity unless they are optimally designed to include revenue-sharing. Gryglewicz et al. (2019) show that tokens are preferable to equity when financing needs and agency conflicts between the entrepreneur and outsiders are not severe. Other studies, such as Li and Mann (2017) and Bakos and Halaburda (2018), focus on the role of tokens in overcoming potential coordination failure among users. Similar to our analysis, Goldstein et al. (2019) also emphasize that tokens can ease the tension between online platforms and customers, although their focus is on monopolistic price discrimination in which tokens unravel monopoly power by serving as durable goods. Mayer (2019) shows that conflicts of interest among the platform developer, users, and speculators interact through token liquidity on utility token platforms where the developer is subject to moral hazard and can sell its retained stake.

Our analysis of the determinants of token prices also contributes to the emerging literature on cryptocurrencies. Many studies focus on the pricing of coins and altcoins, such as Bitcoin, and how the pricing depends on the consensus protocol and its fidelity to network security. Athey et al. (2016) model Bitcoin as a medium of exchange of unknown quality that allows users to avoid bank fees when sending remittances and use the model to guide empirical analysis of the Bitcoin user base. Schilling and Uhlig (2019) study the role of monetary policy in the presence of a cryptocurrency that acts as a private fiat currency. Chiu and Koeppl (2017), Pagnotta and Buraschi (2018), and Pagnotta (2018) develop equilibrium frameworks for Bitcoin with a focus on the interaction between users and miners who foster network security. Cong and He (2019) investigate the trade-off of smart contracts in overcoming adverse selection while also facilitating oligopolistic collusion. Huberman et al. (2019) apply congestion pricing to find the optimal waiting fee structure under the Proof of Work consensus protocol and, in a similar spirit to our analysis, emphasize that decentralization prevents price discrimination by a monopolist. Biais et al. (2018) develop a structural model of cryptocurrency pricing with transactional benefits and costs from hacking and estimate the model with data on Bitcoin; while our paper shares a similar pricing model, we derive a strong network effect in the transactional benefit of the cryptocurrency. Similar to our model, Cong, Li and Wang (2018) also emphasize the strong network effect among platform users by constructing a dynamic model of crypto tokens to study the dynamic feedback between user adoption and the responsiveness of the token price to expectations about future growth of the platform. Our model differs from theirs not only in microfounding the network effect but also by providing a conceptual argument for tokenization.

1 The Model

In this section, we present a baseline model to highlight the key conceptual differences between a token-based platform and an eqity-based platform. There are three dates $t \in \{0, 1, 2\}$. For simplicity, we consider a generic platform, which facilitates bilateral transactions among a group of users. At $t = 0$, the developer of the platform chooses a scheme to fund the platform based on a prior belief about the platform's fundamental, which we will describe in more detail later. At $t = 1$, each potential user chooses whether to join the platform. After joining the platform, a user has the opportunity to randomly match with another user to make mutually beneficial transactions at $t = 1$ and $t = 2$, which can be viewed as the short run and the long run, respectively. In the next section, we will further expand the model to have overlapping generations of users to discuss how resale of tokens may affect the participation decision of each generation of users.

The developer of the platform can choose from two funding schemes for the platform, a conventional equity-based scheme and a token-based scheme. A key feature of our analysis is that the platform owner lacks commitment across the two periods and cannot commit to not abusing the users at $t = 2$ after they have initially joined the platform at $t = 1$. This lack of commitment is a reasonable premise for several reasons. First, it is common for these digital platforms to update their terms of service, which give them the flexibility to adopt strategies that benefit themselves at the expense of the users. Second, digital platforms collect large volumes of user data, which gives the platforms the capacity to take advantage of their users by either selling the data to third parties or by pursuing aggressive advertising strategies. Specifically, we assume that the owner of the platform, which is only present under the equity-based scheme, can take a subverting action at $t = 2$ that monetizes users' private data. Anticipating the owner's lack of commitment may in turn affect the decisions of potential users to join the platform.

At $t = 1$, there is a continuum of potential users with a measure of one unit, indexed by $i \in [0, 1]$. These potential users need to transact goods with each other and can participate in two rounds of trading at $t = 1, 2$ on the platform. To join the platform, each user incurs a personal cost of $\kappa > 0$, which is related to setting up the necessary software and getting familiar with the institutional arrangements of the platform, and may need to pay an entry fee c to the platform. This entry fee may take different forms, depending on the platform's funding scheme, and can be positive or negative. As we will discuss, if the platform is funded by a token-based scheme, a user needs to pay the cost of acquiring a token to join the platform and consequently pay a positive fee. If instead the platform is funded by an equity-based scheme, the owner (i.e., equity holders of the platform) may choose to subsidize each user's initial participation by providing a subsidy, such as giving free digital services. In this case, a user incurs a negative entry fee. Those who do not join initially cannot participate on the platform in either round of transaction. Let $X_i = 1$ if user i joins the platform, and $X_i = 0$ if he chooses not to.

User i is endowed with a certain good, which is distinct from the goods of other users, and has a randomly matched trading partner, user j, in the general pool. Only if both i and j are on the platform, can they trade their goods with each other at $t = 1$ and $t = 2$. After each round of transaction, user i has a Cobb-Douglas utility function over consumption of his own good and the good of user j according to

$$U_i(C_i, C_j) = \left(\frac{C_i}{1-\eta_c}\right)^{1-\eta_c} \left(\frac{C_j}{\eta_c}\right)^{\eta_c}, \tag{1}$$

where $\eta_c \in (0,1)$ represents the weight in the Cobb-Douglas utility function on his consumption of his trading partner's good C_j, and $1 - \eta_c$ is the weight on consumption of his own good C_i. A higher η_c means a stronger complementarity between the consumption of the two goods. Both goods are needed for a user to derive utility from consumption. If one of them is not on the platform, there is no transaction, and each of them gets zero utility. This setting implies that each user cares about the pool of users on the platform, which determines the probability of matching with his trading partner.

User i has a goods endowment of e^{A_i}, which is equally divided across $t = 1$ and $t = 2$. A_i comprises a component A common to all users and an idiosyncratic component:

$$A_i = A + \tau_\varepsilon^{-1/2} \varepsilon_i,$$

with $\varepsilon_i \sim \mathcal{N}(0,1)$ being normally distributed and independent across users and from A. The common component A represents the platform's demand fundamental, and it is publicly observed by all users and the developer only at $t = 1$. At $t = 0$, the developer has a normally distributed prior over A: $A \sim G\left(\bar{A}, \tau_A^{-1}\right)$ and chooses the platform's funding scheme based on this prior belief. We assume that $\int \varepsilon_i d\Phi(\varepsilon_i) = 0$ by the Weak Law of Large Numbers.

The aggregate endowment A is a key characteristic of the platform. A cleverly designed platform serves to amass users with strong needs to transact with each other. As we will show, a higher A leads to more users on the platform, which, in turn, implies a higher probability of each user completing transactions with another user; furthermore, each transaction gives greater surpluses to both parties. One can therefore view A as the demand fundamental of the platform.

When user i is paired with another user j on the platform, we assume that they simply swap their goods, with user i using $\eta_c e^{A_i}$ units of good i to exchange for $\eta_c e^{A_j}$ units of good j. Consequently, both users are able to consume both goods, with user i consuming

$$C_i(i) = (1 - \eta_c) e^{A_i}, \; C_j(i) = \eta_c e^{A_j}, \tag{2}$$

and user j consuming

$$C_i(j) = \eta_c e^{A_i}, \; C_j(j) = (1 - \eta_c) e^{A_j}. \tag{3}$$

We formally derive these consumption allocations between these two paired users in Appendix A through a microfounded trading mechanism between them. As each user receives half of his goods endowment in each period, these consumptions are also equally divided across the two periods. We can use equation (1) to compute the utility surplus $U_{i,1}$ and $U_{i,2}$ of each user on both dates when the transactions happen.

1.1 The Equity-Based Scheme

At $t = 0$, the developer may choose to set up a conventional equity-based scheme to fund the platform. Under this scheme, the developer issues equity, which is fully or partially sold to investors. The developer may also retain some of the equity shares. As it is not crucial to differentiate the heterogeneity between the equity holders, we shall simply refer to them as the owner of the platform.

Owner choices The owner retains not only profit but also control of the platform. The profit motivates the owner to fully build up the platform's user base to maximize its network effect. Specifically, we allow the owner to provide an entry subsidy c (i.e., a negative entry fee) at $t = 1$ and then charge each user a fraction δ of his utility surplus $U_{i,t}$ from the transaction in each period $t = 1, 2$. We impose a cap on the entry subsidy:

$$c \geq -\alpha\kappa.$$

That is, the subsidy cannot be more than a fraction $\alpha \in (0, 1)$ of users' participation cost. As the platform has limited information of the potential users at entry, it cannot discriminate between legitimate users from the relevant pool and opportunistic individuals from outside the relevant pool, who have no intention to participate on the platform but join only to take advantage of the subsidy offered by the platform. Suppose that such opportunistic individuals incur a lower participation cost of $\alpha\kappa$. As a result, any subsidy above $\alpha\kappa$ will attract an arbitrarily large number of opportunistic individuals.

The owner's control of the platform also allows the owner to take a subverting action $s \in \{0, 1\}$ at $t = 2$. That is, if the owner chooses $s = 1$, this action benefits the owner by an amount proportional to the number of users on the platform, $\gamma \int_0^1 X_i di$, at the expense of the users. This action not only prevents any transaction on the platform, but also imposes a utility cost of $\gamma > \alpha\kappa$ on each user.[4] This action can be viewed as a wealth transfer between the owner and users. One can interpret this action as predatory behavior by the owner,

[4]It is convenient, although not essential, to assume the platform collapses for users at date 2. What is needed is that the cost to users, γ, is sufficiently high.

such as the sale of user data to third parties that exploit vulnerable consumers susceptible to temptation goods (Liu, Sockin and Xiong (2020)).

The owner consequently chooses its fees at $t = 1$ to maximize its total expected profit:

$$\Pi^E = \sup_{\{c,\delta,s\}} E\left[\int_0^1 (c + \delta U_{i,1}) X_i di + \int_0^1 ((1-s)\,\delta U_{i,2} + s\gamma) X_i di \mid \mathcal{I}_1\right], \tag{4}$$

where $\mathcal{I}_1 = \{A\}$ is the owner's information set at $t = 1$. For simplicity, we constrain the owner to set the same entry fee c and transaction fee δ for all users, based only on the overall strength of the platform A, which is observed at $t = 1$.[5] The owner chooses its subversive action $s \in \{0, 1\}$ at $t = 2$ to maximize its profit:

$$s = \arg\max \int_0^1 (\delta U_{i,1}(1-s) + \gamma s) X_i di. \tag{5}$$

As the owner's profit is purely driven by the platform fundamental A, the owner's subversive action is also determined by A.

Anticipating the owner's subversive action for certain values of A, potential users are more reluctant to join the platform in this situation. As a result, the owner may prefer precommitting to not subverting at $t = 1$ to maximize the user base. However, such a precommitment is not credible under the equity-based scheme. Even if the owner initially declares its commitment in the platform's charter at $t = 1$, nothing prevents the owner from changing the charter at $t = 2$, just as platforms regularly update their service agreements with users. The token-based scheme allows the platform to precommit to not taking the subversive action because taking such an action requires agreement from the users.

User participation At $t = 1$, each user needs to decide whether to join the platform. We assume that users have quasi-linear expected utility and incur a linear utility gain equal to the total fixed cost of participation $c+\kappa$ if they choose to join the platform at $t = 1$. Furthermore, each user needs to pay a fraction δ of his utility surplus $U_{i,t}$ from any transaction in each period as a variable fee to the platform and may suffer a loss of γ if the owner chooses the subversive action at $t = 2$. In summary, user i makes his participation decision according to

$$\max_{X_i \in \{0,1\}} E\left[(1-\delta)(U_{i,1} + (1-s)\,U_{i,2}) - \kappa - c - \gamma s \mid \mathcal{I}_i\right] X_i, \tag{6}$$

[5] The platform may be able to impose transaction fees that are dependent on each user's transaction need. This flexibility allows the owner to extract more fees from the users, which, in turn, gives the owner an even greater incentive to subsidize user participation. As the owner already chooses the maximum subsidy in our current setting, however, this flexibility does not affect our qualitative comparison of the token-based and equity-based schemes. We prefer our conservative setting for its simplicity.

where $\mathcal{I}_i = \{A, A_i\}$ is the information set of user i at $t = 1$. Note that the expectation of the user's utility flow regards the uncertainty associated with matching a transaction partner. By adopting a Cobb-Douglas utility function with quasi-linearity in wealth, users are risk-neutral with respect to this uncertainty.

It then follows that user i's participation decision is given by

$$X_i = \begin{cases} 1 & \text{if} \quad E\left[(1-\delta)\left(U_{i,1} + (1-s)\,U_{i,2}\right) - \kappa - c - \gamma s \mid \mathcal{I}_i\right] \geq 0 \\ 0 & \text{if} \quad E\left[(1-\delta)\left(U_{i,1} + (1-s)\,U_{i,2}\right) - \kappa - c - \gamma s \mid \mathcal{I}_i\right] < 0 \end{cases}. \tag{7}$$

As the user's expected utility is monotonically increasing with his own endowment, regardless of other users' strategies, it is optimal for each user to use a cutoff strategy. This, in turn, leads to a cutoff equilibrium, in which only users with endowments above a critical level, $\hat{A}^E$, participate in the platform. This cutoff is eventually solved as a fixed point in the equilibrium to equate the fixed participation cost to the expected transaction utility of the marginal user from joining the platform. Given the cutoff strategy for each user who joins the platform if $A_i \geq \hat{A}^E$, a fraction $\Phi\left(\sqrt{\tau_\varepsilon}\left(A - \hat{A}^E\right)\right)$ of users join the platform.

Equilibrium Our model features a rational expectations cutoff equilibrium, which requires the following rational behavior of each user and the owner:

- Owner optimization: The owner chooses a two-part fee structure (c, δ) at $t = 1$ to maximize (4) and chooses its subverting action at $t = 2$ to maximize (5).
- User optimization: Each user chooses X_i at $t = 1$ to solve his maximization problem in (6) for whether to join the platform.

Proposition 1 summarizes the equilibrium under the equity-based scheme.

Proposition 1 *Under the equity-based funding scheme, there is a unique cutoff equilibrium with the following properties:*

1. *If $A > A^E_*$, where the threshold A^* is given by (28), the owner does not subvert the platform at $t = 2$, which in turn leads to the following outcomes at $t = 1$:*
   - *(a) The owner provides the maximum entry subsidy, $c = -\alpha\kappa$;*
   - *(b) The owner sets the transaction fee δ given by (25);*

(c) *Each user i adopts a cutoff strategy to join the platform if A_i is higher than $\hat{A}_{NS}^E$, where $\hat{A}_{NS}^E$ is decreasing in A and is the smaller root of (27).*

2. *If $A \in \left[A_{**}^E, A_*^E\right]$, where A_{**}^E is given by (30), the owner subverts the platform at $t = 2$, which leads to the following outcomes at $t = 1$:*

   (a) *The owner provides the maximum entry subsidy, $c = -\alpha\kappa$;*

   (b) *The owner sets the transaction fee δ given by (26);*

   (c) *Each user i follows a cutoff strategy to join the platform with the cutoff $\hat{A}_{SV}^E$, which is decreasing in A and is the smaller root of (29).*

3. *If $A < A_{**}^E$, the platform breaks down with no user participation at $t = 1$.*

Based on the realization of the demand fundamental A, there are three regions: 1) an equilibrium without subversion when A is higher than A_*^E; 2) an equilibrium with subversion when A is in an intermediate range $\left[A_{**}^E, A_*^E\right]$; and 3) the platform breaks down with no user participation if A is lower than A_{**}^E.

As a result of the network effect, the owner always chooses the maximum entry subsidy, $c = -\alpha\kappa$, to attract the marginal user. As more users join the platform, the greater user base on the platform creates more opportunities for each user to match with another user, which in turn creates more transaction fees for the owner. As the aggregate transaction surplus of the users is increasing and convex with the platform's user base, the marginal revenue from subsidizing the marginal user is always less than the marginal revenue from collecting the transaction fees. With a concentrated ownership in the platform, the owner has the incentive to internalize the network effect and thus to subsidize user participation. This is a key advantage of the conventional equity-based scheme.

However, the concentrated ownership in the platform also creates another problem—the owner may choose to abuse its control power by subverting the platform if the transaction fee is sufficiently low. That is, if the platform fundamental A is lower than a threshold A_*^E, the owner chooses the subversive action at $t = 2$, as described by the second case in Proposition 1. Anticipating the subversion and the resulting damage to the users, potential users are reluctant to join the platform at $t = 1$. Their reluctance forces the owner to reduce the transaction fee, and, despite the reduced fee, the platform participation by the users is still lower than the level in the absence of the subversion. The following proposition establishes this effect induced by the owner's lack of commitment.

Proposition 2 *Under the equity-based scheme, when the subversion equilibrium occurs, that is,* $A \in \left[A^E_{**}, A^E_{*}\right]$*, user participation, owner profit and social surplus all decrease with the degree of user abuse* γ*, while the boundary of platform breakdown* A^E_{**} *increases with* γ*.*

Proposition 2 illustrates that, in the absence of commitment, as γ grows, user participation, owner profit, and social surplus are all lower, and breakdown is more likely to occur. As such, subversion has a negative impact on the ex ante performance of the equity-based scheme. Essentially, the subversion induces another participation cost to users that increases with γ. Thus, the intuition for why entry subsidy is optimal is also the intuition for why owner profit is decreasing in γ. Since the total transaction surplus is greater than the product of the marginal surplus and the size of the user base due to the network effect, there are increasing returns to providing entry subsidy, or, equivalently, decreasing returns to increasing participation cost. This proposition thus highlights that in the presence of the network effect, the lack of commitment is particularly damaging to platforms with relatively weak fundamentals.

1.2 The Token-Based Scheme

The lack of commitment by the platform owner under the conventional equity-based scheme motivates tokenization. The basic motivation is to decentralize the platform so that no one has enough control to take the subversive action. We consider a token-based scheme, which is similar to that of widely used utility tokens. Specifically, this token-based scheme allows the developer to cash out by selling tokens to users at $t = 1$ and, furthermore, delegates the operations of the platform to precoded algorithms, which can be changed only by approval of the token holders. Under this scheme, a user needs to purchase a token to join the platform.[6]

[6] This assumption is consistent with the common practice on many utility token platforms that a user needs to hold tokens in his wallet to complete any bilateral transaction. There are, however, several subtle issues related to this assumption. First, a user may wait to buy a token until immediately before completing a transaction, assuming that market liquidity permits such a timely purchase. As all matched users need to make their transactions at the same time, each has to hold one token at the time of transaction. It follows that requiring each user to hold one token at the time of transaction, instead of when joining the platform, would lead to a quantitatively lower aggregate demand for the token, but would not qualitatively change the key insights of our model. Second, as each user has the need to make one transaction in each period in our model, no one would choose to purchase more than one token; as a result, those who join the platform would each buy one token. Finally, in practice, a user may need to make more than one transaction in a

By acquiring a token at $t = 1$, a user obtains not only the privilege of transacting goods with other users on the platform but also the right to vote on issues related to the platform at $t = 1, 2$. Consequently, utility tokens convey control rights to holders, but do not bestow cash flow rights to the platform's profits like equity. We assume that a majority is needed to pass any decision among the token holders, and that this can be accomplished without conflicts among users. As the token holders would never agree to take the subversive action against themselves, this token-based scheme allows the platform to commit to not taking the subversive action.

This token-based scheme captures the notion of decentralization, which is a key attraction of Bitcoin, and which also underlies many decentralized crypto-based platforms. Decentralization leads to not only the commitment of not abusing users but also to the absence of an owner with ownership in the platform's profit and thus the incentive to subsidize user participation. To the contrary, the marginal user under the token-based scheme needs to pay for the token at entry, in addition to the private participation cost. The lack of entry subsidy implies that the token-based scheme cannot accomplish the first-best social optimum, which requires the maximum user participation. Instead, the token-based scheme serves as a compromise for platforms to precommit to not abusing users. Based on our earlier analysis, such commitment is particularly valuable for platforms with relatively weak fundamentals. This in turn suggests that the token-based scheme is more appealing for platforms with relatively weak fundamentals, a key implication from our later analysis.

For simplicity, we abstract away from several realistic features of crypto-based platforms. First, we do not explicitly model a consensus protocol that determines the right to validate transactions on blockchains, which is present on most existing cryptocurrency platforms.[7] Along this dimension, we do not incorporate miners who work for many crypto-based platforms to complete and record transactions on blockchains based on the Proof of Work protocol, or stakers who stake their wealth to gain priority in completing transactions according to the Proof of Stake protocol. As such, transaction fees in our model accrue to the owner in the equity-based scheme and are wholly absent in the token-based scheme. This allows

period and thus must hold more than one token. Allowing users to have different quantities of transaction needs again may quantitatively change the users' aggregate demand for the token, but not the qualitative implications of our analysis.

[7] We assume the token-based scheme comes with a frictionless consensus protocol that completes all transactions without censorship or charging monopoly markups. See, for instance, Huberman et al (2018) for how Proof of Work decentralized consensus can overcome these issues at the cost of transaction delays.

us to focus on the effects induced by decentralization even in the absence of the frictions associated with implementing a consensus protocol. We will return to these issues in Section 1.4. In addition, we do not allow for retrading of the token in this section, but will extend the model in the next section to allow for retrading by overlapping generations of users.

Developer choice Under the token-based scheme, the developer has a simple choice at $t = 1$ of setting the token price P to maximize his revenue from the token issuance:

$$\Pi^T = \max_P \int_0^1 PX_i(\mathcal{I}_i)\, di,$$

where the token price P adversely affects each user's decision to join the platform. Thus, the developer faces a trade-off between a higher token price and a smaller user base.

User participation Like before, each user chooses at $t = 1$ whether to join the platform by evaluating whether his expected transaction surplus with another matched user on the platform is sufficient to cover the costs of participation and purchasing the token:

$$\max_{X_i \in \{0,1\}} E\left[U_{i,1} + U_{i,2} - \kappa - P \mid \mathcal{I}_i\right] X_i.$$

Under the token-based scheme, the user does not face any subversion risk or transaction fees,[8] but needs to pay the token cost at entry. In the dynamic model introduced in the next section, a user may retrade the token after participating in the platform for a period, and the retrading reduces the token cost in the dynamic setting to the cost of holding the token for one period. Regardless of whether the token can be retraded, a higher token price makes it more costly for potential users to join the platform.

Equilibrium The equilibrium under the token-based scheme is similarly defined as before, with the developer maximizing his revenue and each user making his optimal participation decision. We summarize the equilibrium in the following proposition.

Proposition 3 *Under the token-based funding scheme, the platform breaks down with no user participation if $A < A^T_{**}$, where A^T_{**} is given by (34); and there is a cutoff equilibrium with the following properties if $A \geq A^T_{**}$:*

[8] In practice, some tokenized platforms may require users to pay transaction fees to miners as compensation for their record keeping services. Such transaction fees represent a cost for users to participate on the platform. For simplicity, we abstract transaction fees away from our analysis for simplicity.

1. *Each user i adopts a cutoff strategy in purchasing the token to join the platform:*

$$X_i = \begin{cases} 1 & \text{if } A_i \geq \hat{A}^T \\ 0 & \text{if } A_i < \hat{A}^T \end{cases},$$

where $\hat{A}^T$ is given by the smaller root of (33).

2. *The token price P is given by*

$$P = e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right) - \kappa, \tag{8}$$

where $z^T = \sqrt{\tau_\varepsilon}\left(\hat{A}^T - A\right)$.

As the decentralization instituted by the token-based scheme prevents the platform from taking the subversive action at any time, Proposition 3 confirms that there is no subversion equilibrium. Instead, there is a no-subversion equilibrium if the platform fundamental A is above an equilibrium cutoff A^T_{**}, below which the platform breaks down.

The token price P in (8) is determined by the willingness of the marginal user to participate in the platform. In contrast, the equity price under the equity-based scheme is determined by the transaction fee collected from the average user, who, by the nature of the network effect, benefits more from participation in the platform than the marginal user. This contrast has several important implications. First, token issuance is a less effective funding channel than equity issuance. Second, token prices have different determinants from equity prices and are particularly volatile due to the network effect of the platform. We shall further examine dynamic properties of token prices—determined by the willingness of the marginal user to pay—under the dynamic model setting presented in the next section.

The following proposition compares performance of the token-based scheme along several dimensions to that of the equity-based scheme.

Proposition 4 *Comparing the token-based scheme to the equity-based scheme:*

1. *For a given level of γ, the token-based scheme leads to lower user participation, owner profit, and social surplus if the platform fundamental A is sufficiently high;*

2. *For a given level of A, the token-based scheme leads to higher user participation, owner profit, and social surplus if the degree of user abuse γ is sufficiently high.*

Proposition 4 reflects the trade-off induced by the decentralization of the token-based scheme. On one hand, the decentralization allows the platform to precommit to not abusing users. On the other hand, the decentralization also leads to the absence of any owner with the incentive to subsidize user participation and thus to maximize the network effect. The benefit of the decentralization is greater when the concern about the platform's abuse of users, as measured by the model parameter γ, is sufficiently high, while the benefit from having an owner to subsidize user participation and maximize the network effect is greater when the platform's fundamental is sufficiently strong and the concern about the platform's commitment problem is not severe.

1.3 Choice between Schemes

At $t = 0$, the developer chooses either the equity- or token-based scheme to fund the platform before the platform fundamental A becomes publicly observable at $t = 1$. Instead, the developer makes this choice based on his prior belief distribution $G(A)$ about A. Given the trade-off introduced by the token-based scheme relative to the equity-based scheme, it is intuitive that the developer chooses the token-based scheme when his prior about A is weak, as formally established by the following proposition:

Proposition 5 *Given two possible prior distributions of the developer, G and $\tilde{G}$, such that $G > \tilde{G}$ (in the first-order stochastic dominance sense), the developer is more likely to adopt the token-based scheme under $\tilde{G}$, and the set of priors for which the developer chooses the token-based scheme is increasing in γ. In the special case of a normal prior, $G(A) \sim \mathcal{N}(\bar{A}_G, \tau_A)$, the developer chooses the equity-based scheme if $\bar{A}_G \geq \bar{A}^c(\gamma)$, and the token-based scheme otherwise.*

Proposition 5 shows a sharp implication that the token-based scheme is more likely to be adopted by platforms with relatively weak fundamentals. This implication is consistent with a casual observation that many of the tokenized platforms in recent years tend to be in earlier stages than other traditional equity-based platforms. This stark implication can be directly tested by future empirical studies.

What underlies Proposition 5 is a sharp difference between the equity price and the token price. In the absence of any subversion by the owner of the platform (as in the case in which A is sufficiently strong), the equity price under the equity-based scheme is determined by the aggregate transaction fee collected from all users of the platform. While the transaction surplus is heterogeneous across the pool of users, the aggregate transaction fee is determined by the size of the user pool multiplied by the proportional fee collected from the average user. That is, the equity price is ultimately determined by the transaction surplus of the average user on the platform. In contrast, the token price under the token-based scheme is directly determined by the indifference condition of the marginal user, that is, the token price is equal to the transaction surplus of the marginal user on the platform. In the presence of the network effect, the transaction surplus of the marginal user is lower than that of the average user. This nature of the token price in the token-based scheme makes it less appealing for the developer to raise funding for the platform unless concerns about subversion are severe. Furthermore, the network effect implies that the marginal user of the platform is particularly variable with respect to the platform fundamental and other factors. We later expand our model to a dynamic setting to further analyze the token price dynamics when there is retrade value, which are still determined by the indifference condition of the marginal user.

1.4 Further Discussions

Our analysis adopts a particular form of tokenization to highlight that decentralization through tokenization can lead to a trade-off between commitment of not abusing users and subsidization of user participation. This trade-off emerges because this form of tokenization not only distributes the control rights of the platform to users, which ensures the commitment, but also removes stakeholders with cashflow rights, resulting in the lack of subsidization of participation. To focus on this trade-off, we have abstracted from several other complications in the implementation of tokenization. We now discuss these complications to further argue that the key trade-off highlighted by our analysis is robust to these complicating factors and that the commitment problem associated with centralization of controls may also reappear in different forms under other schemes of tokenization.

Decentralization in practice While we focus our analysis on an archetypal tokenization scheme, which follows the widely observed utility tokens, we recognize that varying degrees of decentralization and tokenization exist in practice. CoinCheckup.com, for instance, classifies the governance structures of blockchain-based platforms into one of four categories, centralized-hierarchical, centralized-flat, semi-centralized, and decentralized, based on the extent to which a platform is governed by its community versus sponsoring organizations or key individuals. Using this classification system, Chen et al. (2020) find a U-shaped relation between the extent of a platform's decentralization and its market capitalization. In addition, other forms of tokens exist beyond utility tokens, such as security tokens, which act like equity but typically do not confer control rights, and governance tokens, which convey control but not cash flow rights.

Consensus protocol Tokenization requires a consensus protocol to maintain the platform's blockchain. Prominent examples of such protocols include Proof of Work, in which miners solve complex computational puzzles to add blocks to the blockchain in exchange for transaction fees and seignorage, and (delegated) Proof of Stake, in which stakers are randomly selected to add blocks based on their staked holdings in exchange for transaction fees. While such protocols have been implemented successfully in practice, they have also created a new set of frictions and conflicts that are absent on conventional platforms. With Proof of Work, miners may have incentive, for instance, to strategically attack the blockchain based on cryptocurrency prices (e.g. Chiu and Koeppl (2017), Budish (2018), Pagnotta (2020)) or fork the blockchain (e.g. Biais et al. (2019), Saleh (2020)), and there are potential economic limits to the scope of its adoption because of congestion (e.g. Huberman et al. (2018), Easley, O'Hara, and Basu (2019), Hinzen et al. (2020)). Furthermore, seignorage on the platform to pay miners acts an inflation tax borne by users and other miners. As a permissioned blockchain, Proof of Stake suffers less from issues of security (e.g. Fanti et al. (2019b), Kose et al. (2020)), but confronts concerns of scalability through the concentration of stake holdings via "richer gets richer" dynamics (e.g. Fanti et al. (2019a), Rosu and Saleh (2020)) and through delegation (e.g. Catalini et al. (2020)).[9] These frictions represent new conflicts between users and record keepers through the implementation of consensus, which potentially harm users or limit their adoption of the platform.

While our model does not incorporate record keepers to specifically analyze the conflicts between users and record keepers, we nevertheless note that giving control rights to record keepers in implementing the consensus protocol may reintroduce the commitment problem in a new form. Since record keepers, such as miners and stakers, have cash flow rights on the platform, earning transaction fees and potentially seignorage, they have incentive to

[9]Technological advances can improve the scalability of consensus protocols, such as the use of off-chain transactions as on Bitcoin's Lightning Network (e.g. Bertucci (2020)). In addition, Ethereum has maximum stake sizes and is introducing sharding to allow for additional decentralization of validation.

concentrate ownership to effectively control the platform.[10] This would de facto reintroduce an owner. Cong, He, and Li (2018), for instance, emphasize that miners have incentive to join mining pools to share risk, yet such pools, in practice, exert significant influence over cryptocurrency platforms. In May 2019, for instance, the BTC.top and BTC.com mining pools, with combined 44% mining power, were criticized for coordinating an "attack" on the BTC Cash blockchain to reverse a hacker's transactions. Similarly, stakers are required to have sizable holdings under the Proof of Stake protocol (at least 32 ETH on Ethereum), which can act as a barrier to entry that enables incumbents to earn rents similar to traditional intermediaries. The presence of record keepers, whose interests need not align with users, can consequently lead to centralization and the potential again for user abuse. Consistent with this concern, Lehar and Parlour (2020) provide evidence that Bitcoin mining pools collude to exploit users for rents in transaction fees.

Optimal design By comparing two specific funding schemes, our analysis also abstracts from the optimal mechanism design given the tension between decentralization and the network effect. Such an exercise would need to be conducted within the context of an optimal implementation protocol for achieving consensus on the blockchain, an issue which is still unsettled in the literature and which may reintroduce the commitment problem as we discussed above. As such, deriving the optimal platform arrangement is beyond the scope of our current analysis.[11]

Our work nevertheless highlights a high-level trade-off that can inform such an optimal design, one that cannot be easily resolved with conventional arrangements for allocating control and cash flow rights. First, we show that tokens are less efficient than equity in extracting value from a platform, as token prices are based on the convenience yield of the marginal user, while equity is based on the average user through the platform's revenue from transaction fees. Second, although users will never act against their own interests by undermining the platform, they also do not have individual incentives to subsidize platform participation, even though it is socially optimal to do so. Third, if tokens carry cash flow rights, in addition to control rights, then users or outsiders may have an incentive to centralize

[10] A related notion is the blockchain trilemma in Abadi and Brunnermeier (2019), which states that it is impossible for a digital record-keeping system to simultaneously be resource efficient, self-sufficient, and rent-free.

[11] The optimal design may also involve a hybrid model of decentralization, such as in Cong, Li, and Wang (2019), in which the platform's owner stewards the platform's operations and development through active token monetary policy.

the platform by amassing tokens, which reintroduces the commitment problem, especially when the token price is low and the platform is vulnerable to subversion.

Our analysis consequently suggests the optimal funding mechanism may mix aspects of the token-based and equity-based schemes, in that it entrusts the platform's operations to users through precoded algorithms that can be modified by the community but entails cash flow rights to the platform's profits under the equity scheme.[12] To see that such a mixed scheme can improve upon the token-based platform, suppose that the platform charges transaction fees as in the equity-based scheme, but these fees are instead paid out to token holders who purchase tokens at price P. It then follows that we can express user i's participation decision as

$$X_i = \begin{cases} 1 \text{ if } E\left[U_{i,1} + U_{i,2} + \delta\left(\frac{U_1+U_2}{\Phi(-z^T)} - U_{i,1} - U_{i,2}\right) - \kappa - P \mid \mathcal{I}_i\right] \geq 0 \\ 0 \text{ if } E\left[U_{i,1} + U_{i,2} + \delta\left(\frac{U_1+U_2}{\Phi(-z^T)} - U_{i,1} - U_{i,2}\right) - \kappa - P \mid \mathcal{I}_i\right] < 0 \end{cases},$$

where U_1 and U_2 are again the total user surpluses at dates 1 and 2, respectively, $\Phi\left(-z^T\right)$ is the number of users on the platform, and $\delta > 0$ is the rate of transaction fees. This mixed scheme provides an indirect subsidy to users with more marginal transaction benefits, for which $\frac{U_1+U_2}{\Phi(-z^T)} - E\left[U_{i,1} + U_{i,2} \mid \mathcal{I}_i\right] > 0$, while taxing users with high transaction benefits, for which $\frac{U_1+U_2}{\Phi(-z^T)} - E\left[U_{i,1} + U_{i,2} \mid \mathcal{I}_i\right] < 0$. It encourages the participation of the marginal user, relative to the token-based scheme that we have examined. Such an arrangement, however, still has lower user participation relative to the equity-based scheme, as the indirect subsidy provided by the mixed scheme cannot reach the same level of subsidy under the equity-based scheme. In addition, and more central to the conflict we study, if tokens convey control rights based on holdings, then users or outsiders may have an incentive to centralize the platform and exploit users by amassing tokens.

Incentives to support the platform In addition to the lack of subsidization for user participation, decentralization may lead to reduced incentives for further development of digital platforms (e..g. Canidio (2018)) or for backstopping them when they require financial support. Even after the successful launch of a platform, it still needs constant innovation and maintenance to compete with other platforms and to further expand its user base. Although a tokenized platform can allocate a certain fraction of its token issuance to reward further

[12] In practice, security tokens can convey cash flow rights but platforms often hesitate to issue them because they are subject to securities regulation. Other tokens, such as the governance token for the DAI stablecoin, Maker, reward holders through a buy and burn program.

platform development, as described in Harvey et al. (2020), tokens are less powerful than equity in creating such incentives. While an owner has a self-interest to support the platform to maximize its profits, individual users and developers require incentives that are difficult to fully specify for any foreseeable contingency in a smart contract. In addition, tokenization may also introduce another class of stakeholders in speculators, whose interests in platform development may conflict with users (e.g. Mayer (2019)).

Conflict among users Tokenization may also help to resolve potential conflicts among users, an issue ignored by our model. Harvey et al. (2020), for instance, discuss how tokens can provide both staked and direct incentives for users to cooperate on a financial service platform through the sophisticated implementation of fees and smart contracts. Severe conflicts among users can also lead to hard forks on a platform that divide the user base, such as on Ethereum after the DAO through the introduction of Ethereum Classic, on Bitcoin over issues of transparency and scalability with SegWit through the introduction of Bitcoin Cash, and on Bitcoin Cash over scalability through the introduction of Bitcoin Satoshi Vision. Furthermore, how users' preferences or private information are aggregated can impact the platform's performance. Tsoukalas and Falk (2020), for instance, study blockchain-based platforms in which decisions are made by token-weighted voting among users, and argue that these schemes are inefficient in aggregating information compared to centralized platforms. Choi and Park (2020) find that decentralization of information production can be socially costly because individual inspectors do not internalize the social benefit of their screening as would a monopolist in the context of academic journals. Such frictions to implementing consensus among users act as a tax on the platform and, to the extent that control rights coincide with cash flow rights, can undermine decentralization and reintroduce the commitment problem that we study.[13]

2 A Dynamic Extension

We now expand our model to a dynamic setting with overlapping generations of users for $t = 0, 1, 2....$ This extension allows us to examine the effects of retrading of tokens. First, how would retrading of tokens affect user participation on the platform? Since users can

[13] When cryptocurrencies have retrade value, voting protocols based on token ownership may also concentrate control among speculators who are often distinct from users.

now resell tokens later, this capital gain defrays some of the initial cost of participation, but it does not substitute for subsidies from an owner. In addition, speculative motives and sentiment may affect the decisions of potential users to participate on the platform. Second, what determines the token's price volatility and expected returns? As the token price is determined by the indifference condition of the marginal user to participate on the platform, token prices may exhibit different dynamics from equity prices. To address these key questions, we focus on the token-based scheme in this dynamic extension.[14]

Users In each period t, there is a pool of potential users, indexed by $i \in [0,1]$. These potential users live for one period and need to transact goods with each other. One may view all users in the setting of the previous section as one generation in this dynamic setting. We consolidate the two rounds of transactions into one round for each generation of users, as we only analyze the token-based scheme for the dynamic setting. Like before, each user chooses whether to purchase a token in order to participate on the platform. Let $X_{i,t} = 1$ if user i purchases the token, and $X_{i,t} = 0$ if he chooses not to purchase. In the next period $t+1$, each user from period t resells his token to future users.

We keep the setting for each generation of users as close as possible to the previous section. When two users are matched on the platform, they trade their goods as specified in (2) and (3), and each of them extracts utility from consuming these two goods according to the Cobb-Douglas utility specified in (1). The goods endowment of user i is $e^{A_{i,t}}$, where $A_{i,t}$ contains a component A_t common to all users of the current generation and an idiosyncratic component $\varepsilon_{i,t}$:

$$A_{i,t} = A_t + \tau_{\varepsilon}^{-1/2}\varepsilon_{i,t},$$

with $\varepsilon_{i,t} \sim \mathcal{N}(0,1)$ being normally distributed and independent of each other, across time, and from A_t. The aggregate endowment A_t fluctuates over time, and follows a random walk

[14] It should be clear that the dynamic setting does not materially change the trade-off between the equity-based and token-based schemes. Under the equity-based scheme, the owner would optimize the franchise value of the platform, V_t^E, over the Bellman Equation:

$$V_t^E = \sup_{\{c,\delta,s\}} E\left[\int_0^1 (c + \delta U_{i,1}) X_i di + \int_0^1 ((1-s)\delta U_{i,2} + s\gamma) X_i di + (1-s) V_{t+1}^E \mid \mathcal{I}_t\right].$$

Under this dynamic setting, the owner may still choose to subvert the platform when its franchise value becomes sufficiently low.

with a constant drift $\mu \in \mathbb{R}$:

$$A_t = A_{t-1} + \mu + \tau_A^{-1/2} \varepsilon_{t+1}^A,$$

where $\varepsilon_{t+1}^A \sim iid\, \mathcal{N}(0,1)$. A higher value of μ means that the platform's demand fundamental grows faster over time.

A potential user i makes his decision to join the platform according to

$$\max_{X_{i,t}} \; (E[U_{i,t} + P_{t+1} \mid \mathcal{I}_{i,t}] - RP_t - \kappa) X_{i,t}, \tag{9}$$

where $\mathcal{I}_{i,t}$ is the information set of user i at date t, and $R \geq 1$ is the interest cost for holding the token for one period. The expectation of the capital gain from holding the token regards the uncertainty in the growth of the platform, while the expectation of the user's utility flow regards the uncertainty associated with matching a transaction partner. By adopting a Cobb-Douglas utility function with quasi-linearity in wealth, users are risk-neutral with respect to the token's capital gain.[15]

Under this dynamic setting, the net cost of holding the token for one period, $RP_t - E[P_{t+1} \mid \mathcal{I}_{i,t}]$, rather than the token price P_t, determines a user's participation cost. We assume that each user knows the value of his own goods endowment $A_{i,t}$ and the current platform fundamental A_t. We also allow all users to receive a public signal Q_t about the next period's innovation to the platform fundamental, ε_{t+1}^A, which by construction is orthogonal to A_t:

$$Q_t = \varepsilon_{t+1}^A + \tau_Q^{-1/2} \varepsilon_t^Q,$$

where $\varepsilon_t^Q \sim iid\, \mathcal{N}(0,1)$. This public signal is similar to a "news" shock in the language of Beaudry and Portier (2006). After observing Q_t, users share the same posterior belief about A_{t+1}, which is normal with the following conditional mean:

$$E[A_{t+1} | \mathcal{I}_t] = A_t + \mu + \frac{\tau_Q}{\tau_\varepsilon + \tau_Q} Q_t.$$

Since Q_t only reveals information about the next period's A_{t+1}, it only impacts users' decisions through their beliefs about the next period's token price, $E[P_{t+1} \mid \mathcal{I}_{i,t}]$, and therefore represents a speculative shock to all of the users. Even though we use the term "user optimism" to denote the speculative shock induced by the public signal Q_t, all of the users in our model are fully rational in information processing.

[15] As Liu and Tsyvinski (2018) find little evidence that cryptocurrencies load on conventional sources of systematic risk, such as market or style factors, such an assumption for the token market is realistic.

Let $\mathcal{I}_t = \sigma\left(\{A_s, P_s, Q_s\}_{s\leq t}\right)$ be the tribe formed by all public information. As each user also observes his own private endowment, $A_{i,t}$, we denote $\mathcal{I}_{i,t} = \sigma\left(\left\{A_{i,t}, \{A_s, P_s, Q_s\}_{s\leq t}\right\}\right)$ as user i's full information set. Like before, it then follows that user i's decision to join the platform is given by

$$X_{i,t} = \begin{cases} 1 & \text{if} \quad E\left[U_{i,t} + P_{t+1} - RP_t \mid \mathcal{I}_{i,t}\right] \geq \kappa \\ 0 & \text{if} \quad E\left[U_{i,t} + P_{t+1} - RP_t \mid \mathcal{I}_{i,t}\right] < \kappa \end{cases}.$$

As the user's expected utility is monotonically increasing with his own endowment, regardless of other users' strategies, it is again optimal for each user to use a cutoff strategy. This, in turn, leads to a cutoff equilibrium in which only users with endowments above a critical level $\hat{A}_t$ buy the token. This cutoff is eventually solved as a fixed point in the equilibrium to equate the token price, net of the expected resale value and participation cost, with the expected transaction utility of the marginal user from joining the platform.

Token supply The supply of tokens, $\Phi(y_t)$, grows over time according to a predetermined schedule:

$$\Phi(y_t) = \Phi(y_{t-1} + \iota),$$

where $\Phi(\cdot)$ is the normal distribution function. This leads to a supply of tokens

$$\Phi(y_t) = \Phi(y_0 + \iota t) \in (0, 1),$$

with y_0 as the supply at the Initial Coin Offering (ICO). This specification captures, as in practice, that the increase in supply tapers over time. For example, the number of new coins and tokens created for Bitcoin and Ethereum periodically halves over time, according to a predetermined schedule, so that the total supply asymptotes to a fixed limit. The token supply continues to grow until it reaches, $\Phi(y_\infty)$, the terminal supply of tokens.

In practice, crypto platforms usually involve a group of miners to verify and record transactions on blockchains. Consistent with this practice, we assume that the seignorage in each period from the scheduled new token issuance, $\Phi(y_{t-1} + \iota) - \Phi(y_{t-1})$, is paid to miners who complete all user transactions in the period. Like before, for simplicity, we do not engage in analyzing more elaborate issues related to miners.

In addition to the new token issuance, there is a continuum of myopic speculators who trade the token to speculate on its price fluctuation over time. Speculators provide liquidity by buying tokens, including those from the old generation of users, and then selling them to the new generation. Through their trading, we assume that the net supply of tokens to users in period t is $\Phi\left(y_t + \lambda_P \log\left(RP_t\right)\right)$, where $\lambda_P \log\left(RP_t\right)$ represents the speculators' token supply in response to the price with $\lambda_P > 0$.[16] When the token price is high relative to the expected future price, the usual downward-sloping demand effect leads to more selling by the speculators and thus a greater token supply to the users. This reduced-form assumption allows us to maintain a similar function form for the equilibrium token price as in the static setting and thus to focus on how various factors of the token price may affect user participation.

Dynamic equilibrium The token market is characterized by the following state variables: the users' demand fundamental A_t, the token supply y_t, and the user optimism driven by the public signal Q_t. We use the notation $\mathcal{I}_t = \{A_t, y_t, Q_t\}$ to represent the state variables at t, which are also equivalent to the set of public information discussed earlier. In each period, users sort into the platform according to a cutoff equilibrium determined by the net benefit of joining the platform, which trades off the opportunity of transacting with other users on the platform and the expected token price appreciation with the cost of participation. The token market equilibrium requires the rational behavior of each user and the clearing of the token market in each period:

- User optimization: Each user chooses $X_{i,t}$ in each period t to solve his maximization problem in (9) for whether to purchase the token.
- In each period, the token market clears:

$$\int_{-\infty}^{\infty} X_{i,t}\left(A_{i,t}, P_t\right) d\Phi\left(\varepsilon_{i,t}\right) = \Phi\left(y_t + \lambda_P \log\left(RP_t\right)\right), \tag{10}$$

  where the user demand is integrated over the idiosyncratic component of their endowments $\{\varepsilon_{i,t}\}_{i\in[0,1]}$.

We characterize the token market equilibrium in the following proposition.

Proposition 6 *The token market equilibrium exhibits the following properties:*

[16] Although we choose this functional form for convenience and tractability, our qualitative insights on the determinants and impact of the user base on token prices and expected returns would remain valid for any upward-sloping supply curve because the marginal user would still price tokens based on his indifference condition.

1. *Each user i follows a cutoff strategy in purchasing the token:*

$$X_{i,t} = \begin{cases} 1 & \text{if} \quad A_{i,t} \geq \hat{A}\left(A_t, y_t, Q_t\right) \\ 0 & \text{if} \quad A_{i,t} < \hat{A}\left(A_t, y_t, Q_t\right) \end{cases},$$

with the cutoff $\hat{A}_t$ solving the following fixed-point condition:

$$\begin{aligned} & e^{(1-\eta_c)\left(\hat{A}_t - A_t\right) + A_t + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - \frac{\hat{A}_t - A_t}{\tau_\varepsilon^{-1/2}}\right) \mathbf{1}_{\{\tau > t\}} + E\left[P_{t+1} | \mathcal{I}_t\right] - \kappa \\ = \; & e^{-\frac{\sqrt{\tau_\varepsilon}}{\lambda_P}\left(\hat{A}_t - A_t\right) - \frac{1}{\lambda_P} y_t}. \end{aligned} \tag{11}$$

2. *The size of the use base on the platform is*

$$\int_{-\infty}^{\infty} X_{i,t}\left(\mathcal{I}_{i,t}\right) d\Phi\left(\varepsilon_{i,t}\right) = \Phi\left(\sqrt{\tau_\varepsilon}\left(A_t - \hat{A}_t\right)\right), \tag{12}$$

and the token price $P\left(A_t, y_t, Q_t\right)$ is determined by

$$P_t = \frac{1}{R}\exp\left(\frac{\sqrt{\tau_\varepsilon}}{\lambda_P}\left(A_t - \hat{A}_t\right) - \frac{1}{\lambda_P} y_t\right). \tag{13}$$

3. *The platform breaks down at τ when the platform fundamental A_t for the first time falls below a critical boundary $A_*\left(y_t, Q_t\right)$, below which equation (11) has no root. The boundary $A_*\left(y_t, Q_t\right)$ moves down with user optimism Q_t and up with user participation cost κ.*

4. *The platform's user base and the token price are both increasing in the platform fundamental A_t and user optimism Q_t.*

Equation (11) provides a fixed-point condition to determine the equilibrium cutoff in each period. The left-hand side of equation (11) reflects the expected benefit to a marginal user with $A_{i,t} = \hat{A}_t$ from acquiring a token to join the platform: the first term is the expected utility flow from transacting with another user on the platform, while the other terms $E\left[P_{t+1} \mid \mathcal{I}_t\right] - \kappa$ represent other benefits, given by the user's expected next-period token price net of the user's participation cost κ. The right-hand side of equation (11) reflects the cost of purchasing a token. This equation may have multiple solutions. When this happens, we assume that all users coordinate on the highest price (i.e., the lowest cutoff) equilibrium in each period, regardless of how many equilibria exist. One can motivate this refinement based on the (dynamic) stability of the potential equilibria.[17]

How does retrading of tokens affect user participation on the platform? We observe from (11) that the effect cost to the marginal user is $\kappa + RP_t - E\left[P_{t+1}|\mathcal{I}_t\right]$. Since the marginal convenience yield (the first term) is positive, it follows that this cost positive. As such, although the ability to retrade the token lowers the effective costs borne by users to join the platform, purchasing tokens still represents a tax on participation rather than a subsidy.

What determines the token price? We iterate forward on (11), imposing transversality (no bubbles), to recover

$$P_t = \sum_{t'=t}^{\infty} E\left[\frac{1}{R^{t'-t}}\left(e^{(1-\eta_c)\left(\hat{A}_{t'}-A_{t'}\right)+A_{t'}+\frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-\frac{\hat{A}_{t'}-A_{t'}}{\tau_\varepsilon^{-1/2}}\right)\mathbf{1}_{\{\tau>t'\}}-\kappa\right)\middle|\mathcal{I}_t\right],$$

which is the discounted present value of future marginal convenience yields. If the developer issues equity against the platform's transaction fees, the price of equity would instead be the present value of the total transaction fees, which is proportional to the average convenience yield of users on the platform. Since the marginal convenience yield is smaller and more sensitive to changes in participation than the average yield, tokens are less powerful in raising financing than equity and also have higher price volatility.

Market breakdown The token market breaks down when the platform fundamental A_t falls below the boundary $A_*\left(y_t, Q_t\right)$. Proposition 6 characterizes the determinants of this boundary. As the participation cost κ makes each user more reluctant to join the platform, the network externality further amplifies this reluctance and raises the breakdown boundary. As the users' speculative motive, driven by their optimism Q_t, helps to overcome the participation externality, the breakdown boundary $A_*\left(y_t, Q_t\right)$ moves down with Q_t. Furthermore, the platform's user base and the token price both rise with Q_t.

To further illustrate the properties of the token market equilibrium, we provide a series

[17] The second (high cutoff) and third (highest cutoff) equilibria may or may not exist at any given date, depending on the expected retrade value of the token. As such, they are dynamically unstable, and we can eliminate them as predictions for the equilibrium outcome. In addition, the second (high cutoff) equilibria is unstable even fixing the token's expected retrade value. Introducing a small amount of noise into users' participation decisions, for instance, and letting this noise become arbitrarily small would ensure convergence away from this second equilibrium to the highest price equilibrium.

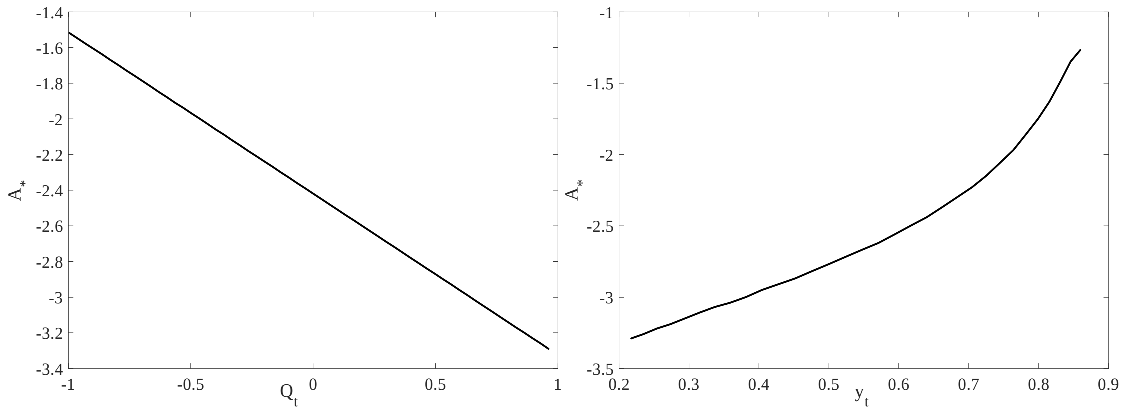

Figure 1: An illustration of the market breakdown boundary A_* with respect to user sentiment Q_t (left panel) and token supply y_t (right panel). Baseline values are given in (14).

of numerical examples based on the following baseline parameter values:

$$\begin{aligned} \mu &= 0.01, \tau_A = 10, y_0 = -0.84, \iota = 0.06, \ \tau_Q = 100, \\ \lambda_P &= 1, \tau_\theta = 1, \eta_c = 0.3, \kappa = 0.03, R = 1.02. \end{aligned} \tag{14}$$

Figure 1 depicts the breakdown boundary $A_*(Q_t, y_t)$ with respect to Q_t (the left panel) and y_t (the right panel). The left panel shows that as user optimism increases, the region of breakdown moves down. The right panel shows that an increase in token supply, by lowering the expected retrade value of the token, increases the breakdown boundary. When the token base is small, there are at least two advantages. First, it is easier to clear markets with a small pool of users. Second, the expected growth of the token value is also higher. As the token supply inflates over time, the effects of token supply imply that the platform becomes more fragile over time, as the token's expected retrade value falls and user participation is driven more by the flow of convenience yield from transactions on the platform.[18]

Life-cycle effects Since our model is nonstationary with the token supply increasing deterministically over time, it has nuanced implications for how platform performance varies

[18] As token supply and user sentiment are not directly related to users' demand for the platform's transaction services, one may view their effects on the token price change in a period as purely nominal. As such, one may argue that users should adjust the number of tokens needed for a transaction to counteract these nominal price effects. We note, however, that nominal rigidity is widely observed in fiat-currency-based monetary systems around the world, possibly because of frictions that prevent timely adjustments in response to price inflation. Similar obstacles also exist for adjusting for nominal price effects on a tokenized platform. For example, such adjustments are challenging because they require a proper decomposition of token price changes in a period into fundamental and nominal factors that, while feasible in our model, is difficult to conduct in practice.

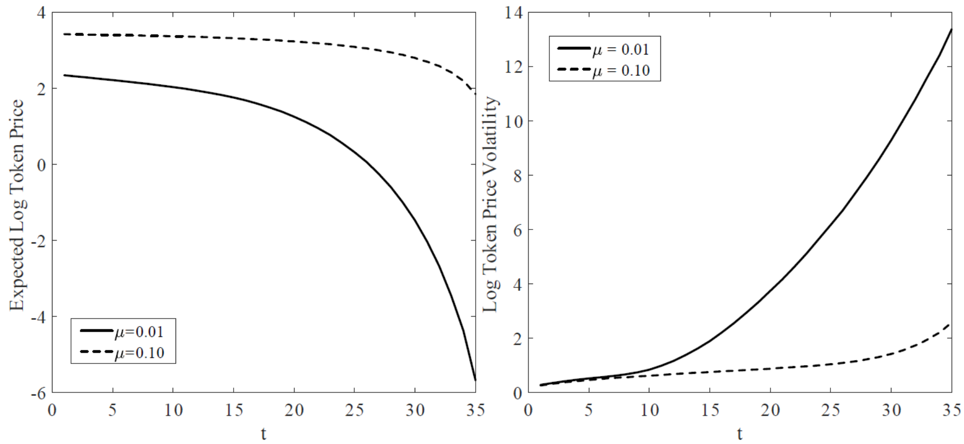

Figure 2: An illustration of life-cycle effects on expected log token price (left panel) and log price volatility (right panel). Baseline values are given in (14).

over the platform's life cycle. Central to understanding these effects is the tension between the contemporaneous convenience yield and the capital gains in each user's total return from holding the token. Since users are risk-neutral, the sum of the two pieces always equal the cost of carry plus the participation cost, $R + \kappa/P_t$, in equilibrium. Thus, when expected future token price appreciation is high, the current convenience yield must be low.

The demand fundamental's expected growth rate μ and the token supply y_t are the two key model parameters that determine the expected token price in the next period. A platform with a higher μ will, on average, see A_t trend upward over time, sustaining a high expected future token price, while a high y_t depresses the expected token price across all values of A_t from supply saturation. The tension between the convenience yield and the expected future token price also impacts the log token price volatility over time. When the fundamental growth rate μ is high, the expected token price remains higher over time. Since more of the token return for high μ platforms is from the capital gains part of the token return, the user base is less sensitive to instantaneous fluctuations in the fundamental. As such, we expect higher μ platforms to have lower token price volatility. In contrast, as the token supply increases, both the region of market breakdown and the importance of the convenience yield in token returns increase, leading to more volatile token prices.

To illustrate these effects, in Figure 2 we consider two platforms that differ only in the expected fundamental growth rate, one with $\mu = 0.01$ and the other with $\mu = 0.10$. To avoid concerns that the patterns are driven by the token supply asymptoting to 1, which covers the full population of users, we assume a maximum token supply of 0.90.[19] Figure 2 shows that when the fundamental growth rate is low ($\mu = 0.01$), the token supply effect dominates, with the expected log token price falling over time while the log token price volatility rising. In contrast, when the fundamental growth rate is high ($\mu = 0.10$), the expected token price declines more slowly over time and log price volatility is more attenuated.

3 Empirical Implications

Our framework highlights a key tension in tokenization—that it provides precommitment at the cost of subsidizing the platform's network effect—and a key characteristic of token prices—that they are determined by the indifference condition of the marginal user. These characteristics distinguish token-based from equity-based platforms and make the dynamics of token prices sharply different from that of equity prices. We now discuss several implications of our model for ICOs and token prices.

Initial coin offerings There is burgeoning empirical literature on ICOs. The key prediction of our framework is that tokenization is appealing for platforms that have ex ante relatively weak fundamentals. Consistent with this observation, Howell et al. (2020), Benedetti and Kostovetsky (2018), and Fisch (2019) document skewed distributions for ICO proceeds in which relatively few ICOs have outsized successes while a significant number fail or raise only modest sums. Benedetti and Kostovetsky (2018) find similar evidence of such skewness when examining token returns prior to secondary market trading on an exchange.[20]

One may also test more directly whether token platforms have relatively weak fundamentals. To do this, one needs to measure the demand fundamental, A_t, of a tokenized platform. Our theory suggests that total transaction fees, which are based on the average convenience yield of users, represents a reliable proxy. Given that many holders of cryptocurrencies may hold them to speculate rather than to use them, measuring platform performance by the number of users or unique wallets may be misleading.

[19] With a fixed token supply less than 1, we must now iterate over a fixed point equation to find the terminal value of the token price and then backwardly solve the model when the supply is less than 0.90.

[20] Admittedly, fear of regulation and potential oversight by the S.E.C. may have impacted the funding decision of entrepreneurs between equity and token financing during this period. While this may have dissuaded some entrepreneurs from issuing tokens, it is not clear that this would impact stronger or weaker projects differentially. In addition, such concerns are less likely to be relevant going forward as the cryptocurrency community continues to establish best practices for transparency of ICOs.

Token price fluctuations We now examine our model's implications for token price fluctuations. Proposition 6 relates the token price to the size of the platform's user base:

$$\log P_t = \frac{1}{\lambda_P} \underbrace{\sqrt{\tau_\varepsilon} \left(A_t - \hat{A}_t \right)}_{\text{size of user base}} - \frac{1}{\lambda_P} y_t,$$

as well as the token supply. As is apparent, the token price positively comoves with the size of the user base, which is consistent with evidence in Bhambhwani et al. (2020). Furthermore, the network effect amplifies fluctuations in the platform's demand fundamental since

$$\frac{d\left(A_t - \hat{A}_t\right)}{dA_t} = 1 - \frac{d\hat{A}_t}{dA_t} > 1,$$

as the endowment of the marginal user, $\hat{A}_t$, is decreasing in A_t from Proposition 6. Shams (2019), for instance, provides evidence that network effects on tokenized platforms amplify the impact of demand shocks on token prices. This amplification further implies that weaker platforms have lower log token prices and higher price volatility because of their highly variable user base, as illustrated in Figure 2, when comparing the two platforms with high and low growth rates (μ). From Figure 2, our model also predicts that log token prices are lower and have higher volatility on more mature platforms, as measured by the extent to which all tokens have been issued, because, all else being equal, token issuance dampens the role of token price appreciation in buoying user participation.

Expected token returns In equilibrium, the expected token return can be expressed as

$$\frac{E\left[P_{t+1} \mid \mathcal{I}_t\right]}{P_t} = \underbrace{R}_{\text{user cost of capital}} - \underbrace{\frac{U_t^*}{P_t}}_{\text{convenience yield}} + \frac{\kappa}{P_t}. \tag{15}$$

Consistent with the empirical findings of Hu, Parlour and Rajan (2018) and Liu and Tsyvinski (2019), the expected excess capital gain in our setting does not exhibit conventional risk premia. The capital gain may still exhibit predictability through the underlying state variables that drive the convenience yield. In our setting, these state variables are the demand fundamental, user optimism, and token supply. Liu and Tsyvinski (2019), for instance, provide evidence that token capital gains are positively related to user sentiment, as measured by either the log ratio between the number of positive and negative phrases of cryptocurrencies in Google searches or the ratio of trading volume to return volatility. Such sentiment may also be systematic across tokens and related to broad optimism or pessimism toward cryptocurrencies as an asset class. Liu and Tsyvinski (2019) show that investor interest in Bitcoin, the most salient cryptocurrency, measured either with Google searches or Twitter post counts, predicts future weekly cryptocurrency price appreciation, and Liu, Tsyvinski and Wu (2019) provide evidence of a Bitcoin market factor that prices the cross-section of cryptocurrencies.

From the decomposition in (15), our model also suggests the participation cost borne by users, which is not directly observed by the econometrician, represents an additional channel of return predictability. As this cost effect is inversely related to the token price and, consequently, market capitalization, our model predicts a size effect in the capital gain of cryptocurrencies. This prediction is consistent with Liu, Tsyvinski and Wu (2019), who find a size factor in the cross-section of cryptocurrency returns, with size measured as either market capitalization, price, or maximum price.

Furthermore, the tension between the net convenience yield $\frac{U_t^* - \kappa}{P_t}$ and capital gains from (15) can lead to autocorrelation in capital gains:

$$Cov\left(\frac{P_{t+2}}{P_{t+1}}, \frac{P_{t+1}}{P_t} \middle| \, \mathcal{I}_{t-1} \right) = -Cov\left(\frac{U_{t+1}^* - \kappa}{P_{t+1}}, \frac{P_{t+1}}{P_t} \middle| \, \mathcal{I}_{t-1} \right) \geq 0,$$

since the token's convenience yield is inversely related to its capital gains.[21] This positive autocorrelation implies token price momentum consistent with the findings of Liu and Tsyvinski (2019), Liu, Tsyvinski and Wu (2019), and Li and Yi (2018).

4 Conclusion

This paper develops a model to examine the decentralization of online platforms through tokenization as an innovation to resolve conflicts of interest between platforms and their users. By delegating control to users through a collection of pre-programmed smart contracts, tokenization acts as a commitment device that prevents a platform from abusing its users. Our analysis highlights that this commitment comes at the cost of not having an owner with an equity stake who would subsidize user participation to maximize the platform's network effect. This cost is present even absent the frictions associated with implementing consensus protocols to accomplish this decentralization, although these frictions can reintroduce the

[21] To derive this, we recognize the innovation $\frac{P_{t+2} - E[P_{t+2} \mid \mathcal{I}_{t+1}]}{P_{t+1}}$ is uncorrelated with information at $t+1$ when users have rational expectations.

conflict between record keepers and users. As such, decentralization through tokenization induces a fundamental trade-off between creating precommitment and subsidizing user participation. As a result of this trade-off, utility tokens are not necessarily ideal for funding all platforms. Instead, utility tokens are more appealing than equity for platforms with weak fundamentals because such platforms tend to have more severe concerns about user abuse. Our analysis also highlights that utility token prices are determined by the marginal user's convenience yield in contrast to equity, whose payoff is determined by the average user. As a result, the price volatility and expected returns of tokens are sharply different from that of equity.

Our analysis sheds light on a key trade-off that can help inform the optimal design of future decentralized platforms and cryptocurrencies. As decentralization offers the promise of disintermediation and a more organic and democratic relationship between users and digital platforms, designing new token and governance architectures that can optimally balance this trade-off is of paramount importance for its success. Such advances could further the objective of empowering users on specialized digital ecosystems.

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Appendix A Microfoundation of Goods Trading

In this Appendix, we microfound the goods trading between two users when they are matched on the platform at date t. As all objects are at date t, we omit time subscripts to economize on notation. We assume that user i maximizes its utility by choosing its consumption demand $\{C_i, C_j\}$ through trading with its trading partner user j subject to its budget constraint:

$$\begin{aligned} U_i &= \max_{\{C_i, C_j\}} U(C_i, C_j; \mathcal{N}) \\ &\quad \text{such that } \; p_i C_i + p_j C_j = p_i e^{A_i}, \end{aligned} \tag{16}$$

where p_i is the price of its good. Similarly, user j solves a symmetric optimization problem for its trading strategy. We also impose market clearing for each user's good between the two trading partners:

$$C_i(i) + C_i(j) = e^{A_i} \quad \text{and} \quad C_j(i) + C_j(j) = e^{A_j}.$$

Furthermore, we assume that users behave competitively and take the prices of their goods as given.

Proposition 7 *User i's optimal good consumptions are*

$$C_i(i) = (1-\eta_c) e^{A_i}, \; C_j(i) = \eta_c e^{A_j},$$

and the price of his good is

$$p_i = e^{\eta_c (A_j - A_i)}.$$

Furthermore, the expected utility benefit of user i at $t = 1$ is given by

$$E\left[U(C_i, C_j) \middle| \mathcal{I}_i\right] = e^{(1-\eta_c)A_i + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} E\left[e^{\eta_c A} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A - \hat{A}}{\tau_\varepsilon^{-1/2}} \right) \middle| \mathcal{I}_i \right],$$

and the ex ante utility benefit of all users before observing their goods endowments is

$$U_0 = e^{A + \frac{1}{2}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}} \Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} + \frac{A - \hat{A}}{\tau_\varepsilon^{-1/2}} \right) \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A - \hat{A}}{\tau_\varepsilon^{-1/2}} \right).$$

Proposition 7 shows that each user spends a fraction $1-\eta_c$ of his endowment on consuming his own good $C_i(i)$ and a fraction η_c on the good of his trading partner $C_j(i)$. The price of each good is determined by its endowment relative to that of the other good. One user's good is more valuable when the other user has a greater endowment, and consequently each user needs to take into account the endowment of his trading partner when making his own decision. The proposition demonstrates that the expected utility of a user in the platform is determined by not only his own endowment e^{A_i} but also the endowments of other users. This latter component arises from the complementarity in the user's utility function.

Appendix B Proofs of Propositions

B.1 Proof of Proposition 1

The expected utility of user i, who chooses to join the platform, to transacting with another user in each round is half of the following:

$$E\left[U_i \mid \mathcal{I}_i,\ A_i,\ \text{matching with user } j\right] = e^{(1-\eta_c)A_i} E\left[e^{\eta_c A_j} \mid \mathcal{I}_i\right],$$

which is monotonically increasing with the user's own endowment A_i. Note that $E\left[e^{\eta_c A_j} \mid \mathcal{I}_i\right]$ is independent of A_i, but dependent on the strategies used by other users. It then follows that user i will follow a cutoff strategy that is monotonic in its own type A_i.

Suppose that every user follows a cutoff strategy with a threshold of $\hat{A}^E$. Then, in each round of transaction, the expected utility of user i from transacting with another user on the platform is half of the following:

$$E\left[U_i \mid \mathcal{I}_i\right] = e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A - \hat{A}^E}{\tau_\varepsilon^{-1/2}}\right). \tag{17}$$

B.1.1 Equilibrium at t = 2

We first examine the equilibrium at $t = 2$. In the absence of subversion, the owner charges a transaction fee δ to complete the transactions of users. Let

$$z^E = \sqrt{\tau_\varepsilon}\left(\hat{A}^E - A\right).$$

Note that the expected fraction of users that participate in the platform is

$$E\left[\int_{-\infty}^{\infty} X_i\left(\mathcal{I}_i\right) d\Phi\left(\varepsilon_i\right) \mid \mathcal{I}_t\right] = \Phi\left(\frac{A_1 - \hat{A}^E}{\tau_\varepsilon^{-1/2}}\right) = \Phi\left(-z^E\right).$$

The owner's profit at $t = 2$ is $\frac{1}{2}\delta U$, where U is the total trade surplus across the two periods, conditional on no subversion:

$$U = e^{A + \frac{1}{2}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^E\right) \Phi\left(\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} - z^E\right).$$

If the owner takes the subversive action, it earns revenue $\gamma\Phi\left(-z^E\right)$. Consequently, the owner takes the subversive action whenever

$$\gamma\Phi\left(-z^E\right) > \frac{1}{2}\delta U \tag{18}$$

and refrains from it otherwise. Consequently, the owner subverts at $t = 2$ whenever the average transaction surplus among users $\delta U / \Phi\left(-z^E\right)$ is sufficiently small. This subversion condition represents an incentive constraint for the platform owner in choosing its fees at $t = 1$, which in turn affects user participation. This condition is eventually determined by the platform fundamental A. Thus, we denote the owner's subversion policy at $t = 2$ by $s(A) \in \{0, 1\}$. As we will show later, the owner will ultimately choose subversion if the platform fundamental A falls below a certain level.

B.1.2 Optimal Fees at t = 1

We now analyze the equilibrium at $t = 1$. We first examine each user's participation choice and the owner's choices of entry and transaction fees by taking the value of A and the owner's subversion policy s as given.

Each user receives two rounds of transaction surplus, after the variable fee δ, if there is no subversion at $t = 2$ and only one round of transaction surplus, and $-\gamma$ otherwise. Given the expression for $E\left[U_{i,1} + U_{i,2} \mid \mathcal{I}_i, A_i = \hat{A}^E\right]$ from (17), the participation constraint for the marginal user with the cutoff endowment $\hat{A}^E$ is

$$\left(1 - \frac{1}{2}s\right)(1-\delta)\, e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^E + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\, \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^E\right) = \kappa + \gamma s + c. \tag{19}$$

The left-hand side is hump-shaped in z^E while the right-hand side has a fixed level at either $\kappa + c$ or $\kappa + \gamma + c$. The right-hand side is positive since $c \geq -\alpha\kappa$, so that the solution cannot be trivial, in which every user enters. This equation has zero or two solutions; when it has two solutions, one is a high cutoff and the other is low. Since user participation and platform revenue are always higher in the low cutoff equilibrium, the platform owner will always coordinate users on the low cutoff equilibrium.

We can then apply the Implicit Function Theorem to recognize that

$$\frac{\partial z^E}{\partial A} = -\frac{1}{(1-\eta_c)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}} < 0, \tag{20}$$

$$\frac{\partial z^E}{\partial \delta} = \frac{1}{1-\delta}\frac{1}{(1-\eta_c)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}} > 0, \tag{21}$$

$$\frac{\partial z^E}{\partial c} = \frac{1}{\left(1 - \frac{1}{2}s\right)(1-\delta)\, e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^E + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\, \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)} \cdot \frac{1}{(1-\eta_c)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}} > 0.$$

The denominator of (20) is positive because it is on left side of the hump. It then follows that

$$\frac{\partial z^E/\partial\delta}{\partial z^E/\partial c} = \left(1-\frac{1}{2}s\right)e^{(1-\eta_c)\tau_\varepsilon^{-1/2}z^E+A+\frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E\right)$$
$$= \left(1-\frac{1}{2}s\right)E\left[U_i \mid \mathcal{I}_i, A_i=\hat{A}^E\right]. \tag{22}$$

We now consider the owner's objective at $t=1$ in choosing its optimal fees:

$$(\delta, c) \in \arg\sup_{\{\delta,c\}} V,$$

where its total profit is

$$V = \frac{1}{2}\delta U + c\Phi\left(-z^E\right) + \max\left\{\frac{1}{2}\delta U, \gamma\Phi\left(-z^E\right)\right\}.$$

The first-order condition for δ is

$$\frac{\partial V}{\partial \delta} = \left(1-\frac{1}{2}s\right)U + \left[\frac{1}{2}\delta\frac{\partial U}{\partial z^E} - c\phi\left(-z^E\right) + \frac{\partial \max\left\{\frac{1}{2}\delta U, \gamma\Phi\left(-z^E\right)\right\}}{\partial z^E}\right]\frac{\partial z^E}{\partial \delta} = 0.$$

The first-order condition for c is

$$\begin{aligned}\frac{\partial V}{\partial c} &= \Phi\left(-z^E\right) + \left[\frac{1}{2}\delta\frac{\partial U}{\partial z^E} - c\phi\left(-z^E\right) + \frac{\partial \max\left\{\frac{1}{2}\delta U, \gamma\Phi\left(-z^E\right)\right\}}{\partial z^E}\right]\frac{\partial z^E}{\partial c} \\ &= \Phi\left(-z^E\right) + \frac{\frac{\partial V}{\partial \delta} - \left(1-\frac{1}{2}s\right)U}{\frac{\partial z^E}{\partial \delta}\Big/\frac{\partial z^E}{\partial c}} \\ &= \Phi\left(-z^E\right) - \frac{U}{E\left[U_i \mid \mathcal{I}_i, A_i=\hat{A}^E\right]},\end{aligned}$$

where we have substituted (22) in the last step. Note that the utility of the marginal user $E\left[U_i \mid \mathcal{I}_i, A_i=\hat{A}^E\right]$ is lower than that of the average user. Thus,

$$\frac{\partial V}{\partial c} < \Phi\left(-z^E\right) - 1 < 0.$$

The owner is constrained in its choice of c and has to choose the lower bound at $c=-\alpha\kappa$.

Given this optimal c, equation (19) reduces to

$$(1-s/2)(1-\delta)e^{(1-\eta_c)\tau_\varepsilon^{-1/2}z^E+A+\frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E\right) = (1-\alpha)\kappa + \gamma s, \tag{23}$$

which identifies $\hat{A}^E$, the smaller root of the above equation when it exists. Comparing the two cases when $s=0$ and $s=1$ for a given level of A and δ, the effective cost to users of joining the platform is higher, leading to a higher participation threshold z^E. Consequently, the owner must charge a smaller δ to attract the same participation when subversion is anticipated. Notice from (19) that $\delta < 1$ since the right-hand side is always nonnegative; users would never pay a cost for zero or negative benefit.

The first-order condition for δ when there is no subversion, given our expression for $\frac{\partial z^E}{\partial \delta}$ and $c = -\alpha\kappa$, becomes

$$(1-\delta)\,U + \frac{\delta \frac{\partial U}{\partial z^E} + \alpha\kappa\phi\left(-z^E\right)}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}} = 0, \tag{24}$$

and, substituting for $\frac{\partial U}{\partial z^E}$, we arrive at

$$\delta = \frac{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)} + \frac{\alpha\kappa\phi\left(-z^E\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E\right)}}. \tag{25}$$

When there is subversion, $s = 1$, then instead

$$\delta = \frac{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E\right)} - \frac{2(\gamma-\alpha\kappa)\phi\left(-z^E\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E\right)}}. \tag{26}$$

Since $\gamma > \alpha\kappa$, by comparing the third term in the numerators of both expressions, it is straightforward to see that δ is higher when there is no subversion for the same A and z^E.

In the next two subsections, we characterize the regions of the platform fundamental A, for which there is and there is no subversion under the optimal fees. We will also consider the possibility of the owner choosing a high-fee level δ at $t = 1$ as a strategy to force no subversion at $t = 2$.

B.1.3 The No-Subversion Equilibrium at t = 1

We now analyze the equilibrium at $t = 1$ when the owner chooses no subversion $s = 0$ at $t = 2$. To avoid confusion, let z^E_{NS} be the equilibrium without subversion and z^E_{SV} be the equilibrium with subversion. We now characterize the domain of A for which a no-subversion equilibrium exists.

Substituting for δ in (25), when there is no subversion, the condition for z^E_{NS} in (23) becomes

$$\begin{aligned}&\frac{\frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E_{NS}\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E_{NS}\right)} + \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E_{NS}\right)} - \alpha\kappa\frac{\phi\left(-z^E_{NS}\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E_{NS}\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^E_{NS}\right)}} e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^E_{NS} + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\,\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^E_{NS}\right) \\ = \;& (1-\alpha)\,\kappa. \end{aligned} \tag{27}$$

The left-hand side of (27) is hump-shaped in z_{NS}^E. To see this, first note that, as $z_{NS}^E \to -\infty$, then the left-hand side tends to 0. As $z_{NS}^E \to \infty$, since $e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{NS}^E + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{NS}^E\right) \to 0$, by L'Hospital's rule and the Sandwich theorem, the left-hand side tends to

$$\begin{aligned}
LHS \quad \to \quad & \lim_{z_{NS}^E \to \infty} 2e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{NS}^E + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{NS}^E\right) \\
& - \frac{\alpha\kappa\phi\left(-z_{NS}^E\right) e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{NS}^E - \frac{1}{2}(1-\eta_c)^2 \tau_\varepsilon^{-1}}}{(1-\eta_c)\tau_\varepsilon^{-1/2}\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right) + \phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right)} \\
= \quad & \lim_{z_{NS}^E \to \infty} - \frac{\alpha\kappa\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right)}{(1-\eta_c)\tau_\varepsilon^{-1/2}\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right) + \phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right)} \\
= \quad & \lim_{z_{NS}^E \to \infty} \alpha\kappa \frac{(1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E}{z_{NS}^E} \\
= \quad & -\alpha\kappa.
\end{aligned}$$

As such, the left-hand side of (27) has finite limits in both tails. We next realize that the optimal δ is a (weakly) decreasing function of z_{NS}^E, $\frac{\partial \delta}{\partial z_{NS}^E} \leq 0$ since the marginal user has a lower endowment, so that $1-\delta$ is (weakly) increasing in z_{NS}^E. Consequently, as a product of a hump-shaped U and (weakly) increasing function $1-\delta$, the left-hand side is hump-shaped in z_{NS}^E. In addition, since $\delta > 0$, it follows that the left-hand side also has a finite upper bound. As such, there are either two or zero solutions to (27). When there are two solutions, the platform owner will always choose the low cutoff solution as it maximizes his revenue.

Notice next that increasing A raises the entire curve on the left-hand side of (27) since $\frac{e^A}{U}$ has no direct dependence on A. Since, in the low cutoff equilibrium, an upward shift in the left-hand side curve lowers the value of z_{NS}^E that intersects $(1-s)\kappa$, we have

$$\frac{dz_{NS}^E}{dA} < 0,$$

in the low cutoff equilibrium, where $\frac{dz_{NS}^E}{dA}$ is the total derivative of z_{NS}^E with respect to A.

Next, when the owner is deciding to subvert, the decision is determined by whether $\frac{1}{2}\delta U$ is greater or less than $\gamma\Phi\left(-z_{NS}^E(A)\right)$. Notice that

$$\begin{aligned}
& \frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z_{NS}^E\right)}\right) \\
= \quad & \frac{1}{\delta U}\frac{d(\delta U)}{dA} + \frac{\phi\left(-z_{NS}^E\right)}{\Phi\left(-z_{NS}^E\right)}\frac{dz_{NS}^E}{dA} \\
= \quad & \frac{1}{\delta}\frac{d\delta}{dA} + 1 - \left(\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z_{NS}^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z_{NS}^E\right)} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{NS}^E\right)} - \frac{\phi\left(-z_{NS}^E\right)}{\Phi\left(-z_{NS}^E\right)}\right)\frac{dz_{NS}^E}{dA}.
\end{aligned}$$

where $\frac{dz^E_{NS}}{dA}$ is again the total derivative of z^E_{NS} with respect to A. Since the hazard function for the normal distribution, $\frac{\phi(-z)}{\Phi(-z)}$, is increasing in z, this implies that both $\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}$ and $\frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}$ are (weakly) greater than $\frac{\phi\left(-z^E_{NS}\right)}{\Phi\left(-z^E_{NS}\right)}$. This, and recalling that $\frac{dz^E_{NS}}{dA} < 0$ imply that

$$\frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z^E_{NS}\right)}\right) > 1 + \frac{1}{\delta}\frac{d\delta}{dA}.$$

Since

$$\begin{aligned}\frac{1}{\delta}\frac{d\delta}{dA} &= \frac{\partial\delta}{\partial A} + \frac{1}{\delta}\frac{\partial\delta}{\partial z^E_{NS}}\frac{\partial z^E_{NS}}{\partial A} \\ &= \frac{-\frac{\alpha\kappa\phi\left(-z^E_{NS}\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)} + \frac{\alpha\kappa\phi\left(-z^E_{NS}\right)}{U}} + \frac{1}{\delta}\frac{\partial\delta}{\partial z^E_{NS}}\frac{\partial z^E_{NS}}{\partial A},\end{aligned}$$

one has that

$$\begin{aligned}\frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z^E_{NS}\right)}\right) &> \frac{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)} + \frac{\alpha\kappa\phi\left(-z^E_{NS}\right)}{U}} + \frac{1}{\delta}\frac{\partial\delta}{\partial z^E_{NS}}\frac{\partial z^E_{NS}}{\partial A} \\ &> \frac{1}{\delta}\frac{\partial\delta}{\partial z^E_{NS}}\frac{\partial z^E_{NS}}{\partial A},\end{aligned}$$

since $(1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z^E_{NS}\right)} \geq 0$ in the low cutoff equilibrium. As argued above, $\frac{\partial\delta}{\partial z^E_{NS}} \leq 0$. Since, in addition $\frac{dz^E_{NS}}{dA} < 0$, it follows that $\frac{\partial\delta}{\partial z^E_{NS}}\frac{\partial z^E_{NS}}{\partial A} > 0$. Therefore,

$$\frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z^E_{NS}\right)}\right) > 0,$$

which implies

$$\frac{d}{dA}\left(\frac{\delta U}{\Phi\left(-z^E_{NS}\right)}\right) > 0.$$

Since there is no subversion when $\frac{\delta U}{\Phi\left(-z^E_{NS}\right)} \geq 2\gamma$, and subversion when $\frac{\delta U}{\Phi\left(-z^E_{NS}\right)} < 2\gamma$, it follows, since $\frac{\delta U}{\Phi\left(-z^E_{NS}\right)}$ is increasing in A, that there exists a critical level A^* such that a no-subversion equilibrium exists if $A \geq A^E_*$, where the unique threshold A^E_* is defined by

$$\frac{\delta\left(A^E_*\right)U\left(A^E_*\right)}{\Phi\left(-z^E_{NS}\left(A^E_*\right)\right)} = 2\gamma. \tag{28}$$

This threshold represents the lowest A for which the owner maximizes his total revenue without subversion.

B.1.4 The Subversion Equilibrium at t = 1

We now analyze the equilibrium at $t = 1$ when the owner chooses subversion $s = 1$ at $t = 2$. In this case, the condition for z_{SV}^E from (23) becomes

$$\begin{aligned}
&\frac{\frac{1}{2}\frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}+\frac{1}{2}\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}+\frac{(\gamma-\alpha\kappa)\phi\left(-z_{SV}^E\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2}+\frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}} e^{(1-\eta_c)\tau_\varepsilon^{-1/2}z_{SV}^E+A+\frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right) \\
= \;& (1-\alpha)\,\kappa+\gamma, \qquad (29)
\end{aligned}$$

where the $\frac{1}{2}$ arises since all $t = 2$ transaction surplus is destroyed by the subversion. Similar to (27), as $z_{NS}^E \to -\infty$, then the left-hand side tends to 0, while, as $z_{NS}^E \to \infty$, the left-hand side tends to $\gamma - \alpha\kappa$. As such, the left-hand side is initially increasing in z_{SV}^E. This equation may have multiple solutions. As before, when this happens, the owner will choose the lowest cutoff, as it gives the highest user participation and revenue. Also similar to (27), an increase in A raises the left-hand side curve, which lowers the equilibrium z_{SV}^E in the lowest cutoff equilibrium. Consequently,

$$\frac{dz_{NS}^E}{dA} < 0,$$

which again is the total derivative of z_{NS}^E with respect to A. In addition, since an increase in z_{NS}^E lowers the endowment of the marginal agent, it follows that $\frac{\partial\delta}{\partial z_{NS}^E} \le 0$.

We next establish the monotonicity of $\frac{\delta U}{\Phi\left(-z_{NS}^E\right)}$ in A when $\delta > 0$. By similar arguments to the no-subversion equilibrium,

$$\begin{aligned}
&\frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z_{NS}^E\right)}\right) \\
=\;& 1+\frac{1}{\delta}\frac{d\delta}{dA}-\left(\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}+\frac{\phi\left((1-\eta_c)\,\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left((1-\eta_c)\,\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}-\frac{\phi\left(-z_{SV}^E\right)}{\Phi\left(-z_{SV}^E\right)}\right)\frac{dz_{NS}^E}{dA} \\
>\;& 1+\frac{1}{\delta}\frac{d\delta}{dA}.
\end{aligned}$$

Since

$$\begin{aligned}
\frac{1}{\delta}\frac{d\delta}{dA} &= \frac{\partial\delta}{\partial A}+\frac{1}{\delta}\frac{\partial\delta}{\partial z_{SV}^E}\frac{\partial z_{SV}^E}{\partial A} \\
&= \frac{\frac{2(\gamma-\alpha\kappa)\phi\left(-z_{SV}^E\right)}{U}}{(1-\eta_c)\,\tau_\varepsilon^{-1/2}-\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2}-z_{SV}^E\right)}-\frac{2(\gamma-\alpha\kappa)\phi\left(-z_{SV}^E\right)}{U}}+\frac{1}{\delta}\frac{\partial\delta}{\partial z_{SV}^E}\frac{\partial z_{SV}^E}{\partial A},
\end{aligned}$$

it follows that

$$\begin{aligned}
&\frac{d}{dA}\log\left(\frac{\delta U}{\Phi\left(-z_{SV}^{E}\right)}\right) \\
> \quad &\frac{\left(1-\eta_c\right)\tau_{\varepsilon}^{-1/2}-\frac{\phi\left(\eta_c\tau_{\varepsilon}^{-1/2}-z_{SV}^{E}\right)}{\Phi\left(\eta_c\tau_{\varepsilon}^{-1/2}-z_{SV}^{E}\right)}}{\left(1-\eta_c\right)\tau_{\varepsilon}^{-1/2}-\frac{\phi\left(\eta_c\tau_{\varepsilon}^{-1/2}-z_{SV}^{E}\right)}{\Phi\left(\eta_c\tau_{\varepsilon}^{-1/2}-z_{SV}^{E}\right)}-\frac{2(\gamma-\alpha\kappa)\phi\left(-z_{SV}^{E}\right)}{U}}+\frac{1}{\delta}\frac{\partial\delta}{\partial z_{SV}^{E}}\frac{\partial z_{SV}^{E}}{\partial A} \\
> \quad &\frac{1}{\delta}\frac{\partial\delta}{\partial z_{SV}^{E}}\frac{\partial z_{SV}^{E}}{\partial A}.
\end{aligned}$$

As argued above, $\frac{\partial\delta}{\partial z_{SV}^{E}}\leq 0$. Since $\frac{dz_{NS}^{E}}{dA}<0$, it follows that $\frac{\partial\delta}{\partial z_{SV}^{E}}\frac{\partial z_{SV}^{E}}{\partial A}>0$. Therefore,

$$\frac{d}{dA}\left(\frac{\delta U}{\Phi\left(-z_{SV}^{E}\right)}\right)>0.$$

Consequently, there exists a critical A_{*c}^{E} such that subversion occurs for $A\leq A_{*c}^{E}$, where A_{*c}^{E} satisfies

$$\frac{\delta U\left(A_{*c}^{E}\right)}{\Phi\left(-z_{SV}^{E}\left(A_{*c}^{E}\right)\right)}=2\gamma.$$

Suppose now that for a given level of A, both a subversion and a no-subversion equilibrium exist, that is, solutions to both (27) and (29) exist. In the equilibrium without subversion

$$\frac{1}{2}\frac{\delta\left(z_{NS}^{E}\right)U\left(z_{NS}^{E}\right)}{\Phi\left(-z_{NS}^{E}\right)}\geq\gamma,$$

while in the equilibrium with subversion

$$\gamma\geq\frac{1}{2}\frac{\delta\left(z_{SV}^{E}\right)U\left(z_{SV}^{E}\right)}{\Phi\left(-z_{SV}^{E}\right)},$$

which implies that

$$\frac{\delta\left(z_{NS}^{E}\right)U\left(z_{NS}^{E}\right)}{\Phi\left(-z_{NS}^{E}\right)}\geq\frac{\delta\left(z_{SV}^{E}\right)U\left(z_{SV}^{E}\right)}{\Phi\left(-z_{SV}^{E}\right)}.$$

Since $\frac{\delta(z)U(z)}{\Phi(-z)}$ is monotonically decreasing in z, it follows that $z_{NS}^{E}\leq z_{SV}^{E}$, and user participation is higher in the equilibrium without subversion. It then follows that

$$\begin{aligned}
\delta\left(z_{NS}^{E}\right)U\left(z_{NS}^{E}\right)-\Phi\left(-z_{NS}^{E}\right)\alpha\kappa \quad &> \quad \frac{1}{2}\delta\left(z_{NS}^{E}\right)U\left(z_{NS}^{E}\right)+\Phi\left(-z_{NS}^{E}\right)\gamma-\Phi\left(-z_{NS}^{E}\right)\alpha\kappa \\
&> \quad \frac{1}{2}\delta\left(z_{SV}^{E}\right)U\left(z_{SV}^{E}\right)+\Phi\left(-z_{SV}^{E}\right)\left(\gamma-\alpha\kappa\right).
\end{aligned}$$

As such, when both equilibria exist, the no-subversion equilibrium generates a higher profit for the owner. As such, the owner will choose not to subvert even when subverting is a sustainable action. Consequently, the cutoff A_*^E is the relevant cutoff for separating the equilibria with and without subversion.

Next, note that the left-hand side of (29), which we define as $LHS\left(z_{SV}^E\right)$, is hump-shaped in z_{SV}^E. Thus, it achieves its maximum at an interior point $\bar{z}(A) = \sup_z LHS(z)$. As this peak is increasing in A, it follows that there exists a critical A_{**}^E, such that

$$LHS\left(\bar{z}\left(A_{**}^E\right)\right) = (1-\alpha)\kappa + \gamma. \tag{30}$$

Thus, an equilibrium with subversion exists when $A \geq A_{**}^E$ and does not exist otherwise.

One may be concerned that the region $\left[A_{**}^E, A_*^E\right]$ may be an empty set for a certain value of γ. Suppose that this is the case. That is, as A decreases from ∞ to 0, the equilibrium shifts from no-subversion equilibrium to no equilibrium at A_*^E. As the owner is willing to subsidize participation as long as there is a positive profit. Thus, it must be

$$V\left(A_*^E\right) = \delta U - \alpha\kappa\Phi\left(-z_{NS}^E\right) = 0,$$

which implies that $\delta U = \alpha\kappa\Phi\left(-z_{NS}^E\right)$. Since $\gamma > \alpha\kappa$, we have

$$\frac{1}{2}\delta U = \frac{1}{2}\alpha\kappa\Phi\left(-z_{NS}^E\right) < \gamma\Phi\left(-z_{NS}^E\right).$$

It follows that the owner is better off by taking the subversive action in this case. Thus, a subversion equilibrium exists. Thus, the region $\left[A_{**}^E, A_*^E\right]$ cannot be empty.

B.1.5 Forcing Equilibrium at t = 1

One may argue that the owner may internalize his lack of commitment by treating the subversion condition as an incentive constraint. That is, the owner can avoid subverting the platform by imposing a constraint to prevent the subversion condition in (18) from being satisfied at $t = 2$. We now examine this possibility by constraining the owner's choice of δ at $t = 1$ such that $\frac{\delta U}{\Phi(-z^E)} \geq 2\gamma$ (i.e., the owner will not choose subversion at $t = 2$). This condition imposes a lower bound on δ: $\delta \geq \underline{\delta} = \frac{2\gamma\Phi\left(-z^E\right)}{U}$.

Suppose that when this constraint is not imposed, there is a subversion equilibrium with δ_{SV} as the transaction fee and z_{SV}^E as the participation cutoff, and that when this constraint is imposed, there is a different forcing equilibrium with $\underline{\delta}$ as the transaction fee and $z_{forcing}^E$ as the participation cutoff. It is important to note that $\underline{\delta}$ is always in the owner's choice set. As such, it must give a lower profit to the owner than δ_{SV}. That is, $V\left(\underline{\delta}, z_{forcing}^E\right) < V\left(\delta_{SV}, z_{SV}^E\right)$, which implies that the forcing equilibrium is dominated by the subversion equilibrium if both exist and are different.

Furthermore, if a forcing equilibrium with $\underline{\delta}$ exists and if no subversion equilibrium exists, then the owner would choose $\underline{\delta}$ even without the constraint. Taken together, there is no need to separately consider the forcing equilibrium.

B.1.6 Equilibrium Uniqueness

As we discussed at the beginning of this proof, it is optimal of each user to adopt a cutoff strategy because his expected utility from joining the platform is monotonically increasing with his own good endowment. The uniqueness of the equilibrium follows directly from the platform owner's choice of the lowest cutoff and thus highest profit equilibrium, if there are multiple equilibria that are feasible.

B.2 Proof of Proposition 2

First, we first establish that the cutoff for the marginal user, z_{SV}^E, is decreasing with γ in the subversion equilibrium. Applying the Implicit Function theorem to (29), we have that

$$\frac{dz_{SV}^E}{d\gamma} = -\frac{\dfrac{\frac{\phi\left(-z_{SV}^E\right)}{U} e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{SV}^E + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{(1-\eta_c)\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}} - 1}{\frac{dLHS}{z_{SV}^E}},$$

where LHS is the left-hand side of (29). From the proof of Proposition 1, $\frac{dLHS}{z_{SV}^E}$ is positive in the lowest cutoff equilibrium. Furthermore, with some manipulation, one has that

$$\begin{aligned}
&\frac{\frac{\phi\left(-z_{SV}^E\right)}{U} e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{SV}^E + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}} - 1 \\
= &\frac{\frac{\phi\left(-z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)} e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z_{SV}^E - \frac{1}{2}(1-\eta_c)^2\tau_\varepsilon^{-1}}}{\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}} - 1 \\
= &\frac{\frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}}{\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z_{SV}^E\right)}} - 1 < 0.
\end{aligned}$$

Consequently, it follows that $\frac{dz_{SV}^E}{d\gamma} > 0$.

Since z_{SV}^E is increasing in γ, it follows that the participation constraint for the marginal user is tightening in γ. As such, from (30), it follows that the critical A_{**}^E at which breakdown occurs with subversion is also increasing in γ.

Second, owner profit in the subversion equilibrium being decreasing in γ follows from the envelope condition on fees, δ, that

$$\frac{dV}{d\gamma} = \left[\frac{1}{2}\delta \frac{\partial U}{\partial z_{SV}^E} - (\gamma - \alpha\kappa)\,\phi\left(-z_{SV}^E\right)\right] \frac{\partial z_{SV}^E}{\partial \gamma} + \Phi\left(-z_{SV}^E\right).$$

Applying the Implicit Function theorem to (23), it follows that

$$\frac{\partial z_{SV}^{E}}{\partial \gamma} = \frac{1}{\gamma} \frac{1}{\left(1-\eta_c\right)\tau_{\varepsilon}^{-1/2} - \frac{\phi\left(\eta_c \tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)}{\Phi\left(\eta_c \tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)}},$$

and, by using (21), we can express $\frac{\partial z_{SV}^{E}}{\partial \gamma}$ as

$$\frac{\partial z_{SV}^{E}}{\partial \gamma} = \frac{1-\delta}{\gamma} \frac{\partial z_{SV}^{E}}{\partial \delta} > 0.$$

Substituting this expression into $\frac{dV}{d\gamma}$, we arrive at

$$\frac{dV}{d\gamma} = \left[\frac{1}{2}\delta \frac{\partial U}{\partial z_{SV}^{E}} - \left(\gamma - \alpha\kappa\right)\phi\left(-z_{SV}^{E}\right)\right] \frac{\partial z_{SV}^{E}}{\partial \delta} \frac{1-\delta}{\gamma} + \Phi\left(-z_{SV}^{E}\right).$$

Substituting now the first-order necessary condition for the optimal choice of δ from (21) into $\frac{dV}{d\gamma}$, it follows that

$$\gamma \frac{dV}{d\gamma} = \gamma \Phi\left(-z_{SV}^{E}\right) - \frac{1}{2}\left(1-\delta\right) U.$$

Note that for all users that join the platform, it must be the case that

$$\frac{1}{2}\left(1-\delta\right) e^{\left(1-\eta_c\right)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_{\varepsilon}^{-1}} \Phi\left(\eta_c \tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right) \geq \left(1-\alpha\right)\kappa + \gamma,$$

which by integrating both sides against the population density of users, $\phi\left(\sqrt{\tau_{\varepsilon}}\left(A_i - A\right)\right)$ for $A_i \geq \hat{A}_{SV}^{E}$, we arrive at

$$\frac{1}{2}\left(1-\delta\right) U \geq \left(\left(1-\alpha\right)\kappa + \gamma\right)\Phi\left(-z_{SV}^{E}\right), \tag{31}$$

and therefore

$$\gamma \frac{dV}{d\gamma} \leq \gamma \Phi\left(-z_{SV}^{E}\right) - \left(\left(1-\alpha\right)\kappa + \gamma\right)\Phi\left(-z_{SV}^{E}\right) = -\left(1-\alpha\right)\kappa \Phi\left(-z_{SV}^{E}\right) < 0.$$

Therefore, $\frac{dV}{d\gamma} < 0$.

Finally, note that total social surplus with subversion is given by

$$U_0 = \frac{1}{2}U - \kappa \Phi\left(-z_{SV}^{E}\right),$$

since all surplus at $t = 2$ is destroyed by subversion. It is straightforward to verify that

$$\frac{2}{U}\frac{dU_0}{dz_{SV}^{E}} = \frac{2\kappa \Phi\left(-z_{SV}^{E}\right)}{U} \frac{\phi\left(-z_{SV}^{E}\right)}{\Phi\left(-z_{SV}^{E}\right)} - \frac{\phi\left(\eta_c \tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)}{\Phi\left(\eta_c \tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)} - \frac{\phi\left(\left(1-\eta_c\right)\tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)}{\Phi\left(\left(1-\eta_c\right)\tau_{\varepsilon}^{-1/2} - z_{SV}^{E}\right)}.$$

When the social surplus is nonnegative, it follows that $\frac{2\kappa\Phi\left(-z_{SV}^{E}\right)}{U} \leq 1$, and consequently

$$\frac{2}{U}\frac{dU_0}{dz_{SV}^{E}} < \frac{\phi\left(-z_{SV}^{E}\right)}{\Phi\left(-z_{SV}^{E}\right)} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{SV}^{E}\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_{SV}^{E}\right)} - \frac{\phi\left(\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} - z_{SV}^{E}\right)}{\Phi\left(\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} - z_{SV}^{E}\right)} < 0,$$

since the hazard function for the normal distribution, $\frac{\phi(-x)}{\Phi(-x)}$, is increasing in x. Consequently, when the social surplus is positive, it is increasing in user participation.

Note now from (31) that

$$\gamma\Phi\left(-z_{SV}^{E}\right) \leq \frac{1}{2}\left(1-\delta\right)U - \left(1-\alpha\right)\kappa\Phi\left(-z_{SV}^{E}\right),$$

from which it follows that

$$V = \frac{1}{2}\delta U + \left(\gamma - \alpha\kappa\right)\Phi\left(-z_{SV}^{E}\right) \leq \frac{1}{2}U - \kappa\Phi\left(-z_{SV}^{E}\right),$$

so that the owner's total profit is (weakly) less than the social surplus from the initial period. Consequently, subversion destroys the owner's profit at $t = 2$ and delivers, at best, the total surplus at date 1. Furthermore, if the social surplus is negative, then $V < 0$. Consequently, if the platform operates, it must be the case that $U_0 \geq 0$, and therefore the social surplus is decreasing in z_{SV}^{E}. As $\frac{dz_{SV}^{E}}{d\gamma} > 0$, it follows that the social surplus is also decreasing in γ.

B.3 Proof of Proposition 3

We first examine the decision of a user to purchase the token. The expected utility of user i, who chooses to join the platform at $t = 1$ and then transact with another user at $t = 1$ and $t = 2$, is

$$E\left[U_{i,t} \mid \mathcal{I}_i,\ A_i,\ \text{matching with user } j\right] = \frac{1}{2}e^{(1-\eta_c)A_i}E\left[e^{\eta_c A_j} \mid \mathcal{I}_i\right],$$

which is monotonically increasing with the user's own endowment A_i. Note that $E\left[e^{\eta_c A_j} \mid \mathcal{I}_i\right]$ is independent of A_i but dependent on the strategies used by other users. It then follows that user i will adopt a cutoff strategy that is monotonic in his own type A_i.

Suppose that every user uses a cutoff strategy with a threshold of $\hat{A}^T$. Then, the expected utility of user i at $t \in \{1, 2\}$ is

$$E\left[U_{i,t}|\mathcal{I}\right] = \frac{1}{2}e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}}\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - \sqrt{\tau_\varepsilon}\left(\hat{A}^T - A\right)\right),$$

Since each user's endowment is the same in both periods, each user receives $E\left[U_i|\mathcal{I}\right] = E\left[U_{i,1} + U_{i,2}|\mathcal{I}\right]$ in total.

If a potential user does not join the platform, he saves the participation and token costs, $\kappa + P$. Consequently, we require that the expected utility of users from joining the platform at $t = 1$ exceeds $\kappa + P$. Consider a user with the critical endowment $A_i = \hat{A}^T$. His indifference condition to joining the platform is

$$E\left[U_{i,1} + U_{i,2}|\mathcal{I}, A_i = \hat{A}^T\right] = e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right) = \kappa + P, \qquad (32)$$

where $z^T = \sqrt{\tau_\varepsilon}\left(\hat{A}^T - A\right)$.

Note, by the Implicit Function theorem, that

$$\frac{\partial z^T}{\partial P} = \frac{1}{\left((1-\eta_c)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)}\right) e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)} > 0$$

since the denominator is positive in the low cutoff equilibrium. As before, we assume that, if there are two solutions for z^T, the owner will coordinate users on the low cutoff (high price) equilibrium, as opposed to the high cutoff (low price) equilibrium, since both user participation and owner profit are higher in this equilibrium.

For any other user whose endowment satisfies $A_i > \hat{A}^T$, notice that

$$\begin{aligned} E\left[U_{i,1} + U_{i,2}|\mathcal{I},\right] &= e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A - \hat{A}^T}{\tau_\varepsilon^{-1/2}}\right) \\ &> e^{(1-\eta_c)\tau_\varepsilon^{-1/2}\hat{A}^T + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A - \hat{A}^T}{\tau_\varepsilon^{-1/2}}\right) \\ &= \kappa + P, \end{aligned}$$

and consequently it is optimal for users to follow a cutoff strategy in which users with $A_i \geq \hat{A}^T$ join, and users with $A_i < \hat{A}^T$ do not.

Since $A_i = A + \varepsilon_i$, it then follows that a fraction $\Phi\left(-\sqrt{\tau_\varepsilon}\left(\hat{A}^T - A\right)\right)$ of the users enter the platform, and a fraction $\Phi\left(\sqrt{\tau_\varepsilon}\left(\hat{A}^T - A\right)\right)$ choose not to participate. It is the integral over the idiosyncratic endowment of users ε_i that determines the fraction of potential users on the platform. The owner consequently maximizes

$$\Pi^T = P\Phi\left(-z^T\right),$$

which is the revenue from the sale of tokens, specifically, the price P multiplied by the quantity $\Phi\left(-z^T\right)$. The first-order condition with respect to the price, P, is

$$\Phi\left(-z^T\right) - P\phi\left(-z^T\right)\frac{\partial z^T}{\partial P}\begin{cases} = 0 \text{ if } P > 0 \\ < 0 \text{ if } P = 0 \end{cases}.$$

Substituting with $\frac{\partial z^T}{\partial P}$, an interior solution for the token price, when it exists, is given by

$$P = \frac{\Phi\left(-z^T\right)}{\phi\left(-z^T\right)}\left(\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right)}\right) e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right) \geq 0.$$

Notice that the hazard rate $\phi\left(-z^T\right)/\Phi\left(-z^T\right)$ is increasing in z^T. As such, P decreases from ∞ to 0, at which point the nonnegativity constraint imposes a critical $\bar{z}^T$ such that

$$\frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - \bar{z}^T\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - \bar{z}^T\right)} = \left(1-\eta_c\right)\tau_\varepsilon^{-1/2},$$

above which the token price is fixed at a corner solution of 0. This corner corresponds to the peak of the hump of $e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right)$.

Equating the two representations for P, we arrive at

$$\left(1 - \frac{\Phi\left(-z^T\right)}{\phi\left(-z^T\right)}\left(\left(1-\eta_c\right)\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right)}\right)\right) e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - z^T\right) = \kappa, \tag{33}$$

which identifies $z^T \leq \bar{z}^T$. The left-hand side of (33) is increasing from $-\infty$ to $\bar{z}^T$, with a peak at $\bar{z}^T$, while the RHS is fixed at κ. Suppose

$$e^{(1-\eta_c)\tau_\varepsilon^{-1/2} \bar{z}^T + A + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - \bar{z}^T\right) \geq \kappa.$$

Then, there exists a cutoff equilibrium with the cutoff given by (33). If instead

$$e^{(1-\eta_c)\tau_\varepsilon^{-1/2} \bar{z}^T + A_1 + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - \bar{z}^T\right) < \kappa,$$

then the LHS of (33) never intersects the RHS, and consequently, there is no equilibrium.

Note that the LHS of (33) is monotonically increasing in the platform fundamental A. As such, there exists a critical A_{**}^T such that

$$e^{(1-\eta_c)\tau_\varepsilon^{-1/2} \bar{z}^T\left(A_{**}^T\right) + A_{**}^T + \frac{1}{2}\eta_c^2\tau_\varepsilon^{-1}} \Phi\left(\eta_c\tau_\varepsilon^{-1/2} - \bar{z}^T\left(A_{**}^T\right)\right) = \kappa. \tag{34}$$

There exists an equilibrium with a nonnegative profit for the owner if $A \geq A_{**}^T$, and such an equilibrium does not exist otherwise.

B.4 Proof of Proposition 4

Our comparison of the two platform funding schemes will eventually simplify to the observation that the token-based scheme represents a constrained revenue optimization (only a fixed fee) compared to the equity-based scheme.

We begin with the case of no subversion under the equity-based scheme and compare user participation. Suppose that A is sufficiently high so that there is no subversion, $A \geq A_*^E$. We begin with user participation. We first recognize, from Proposition 1, that we can express (23) when there is no subversion as

$$e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^E + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^E\right) = \kappa + p,$$

where

$$p = \frac{(1-\alpha)\,\kappa}{1-\delta} - \kappa$$

is the implicit token price of participation on the platform. From Proposition 3, (8) reveals that

$$e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right) = \kappa + P.$$

Now consider a perturbation of the owner's profit on the token platform, Π^T, with respect to the participation cutoff z^T :

$$\begin{aligned}
&\frac{1}{\phi\left(-z^T\right)} \frac{d\Pi^T}{dz^T} \\
= \;& \frac{\Phi\left(-z^T\right)}{\phi\left(-z^T\right)} \left((1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)} \right) e^{(1-\eta_c)\tau_\varepsilon^{-1/2} z^T + A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right) - P,
\end{aligned}$$

which at the equity-based scheme cutoff z^E where $P = p$ reduces, with some manipulation, to

$$\begin{aligned}
H\left(z^T\right) &= \frac{1-\delta}{(1-\alpha)\,\kappa\phi\left(-z^T\right)} \frac{dV^T}{dz^T}\Big|_{z^T = z^E} \\
&= \frac{\Phi\left(-z^T\right)}{\phi\left(-z^T\right)} \left((1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)} \right) + \frac{\alpha - \delta}{1-\alpha},
\end{aligned}$$

where δ is given by Proposition 1. Substituting with the FOC for δ, (24), we recognize that

$$\frac{\alpha-\delta}{1-\alpha} = -1 - \frac{\delta\left(\frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^T\right)} + \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^Y\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z^Y\right)} \right) + \frac{\alpha\kappa\phi\left(-z^Y\right)}{U}}{(1-\alpha)\left((1-\eta_c)\,\tau_\varepsilon^{-1/2} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^E\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z^E\right)} \right)} < 0$$

since $\delta > 0$, we arrive at

$$H\left(z^{T}\right) = \frac{\Phi\left(-z^{T}\right)}{\phi\left(-z^{T}\right)}\left(\left(1-\eta_{c}\right)\tau_{\varepsilon}^{-1/2} - \frac{\phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2} - z^{T}\right)}{\Phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2} - z^{T}\right)}\right) - 1$$
$$-\frac{\delta\left(\frac{\phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2}-z^{T}\right)}{\Phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2}-z^{T}\right)} + \frac{\phi\left((1-\eta_{c})\tau_{\varepsilon}^{-1/2}-z^{Y}\right)}{\Phi\left((1-\eta_{c})\tau_{\varepsilon}^{-1/2}-z^{Y}\right)}\right) + \frac{\alpha\kappa\phi\left(-z^{Y}\right)}{U}}{(1-\alpha)\left(\left(1-\eta_{c}\right)\tau_{\varepsilon}^{-1/2} - \frac{\phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2}-z^{T}\right)}{\Phi\left(\eta_{c}\tau_{\varepsilon}^{-1/2}-z^{T}\right)}\right)}.$$

When $H > 0$, then $z^{T} > z^{E}$, and participation is higher under the equity-based scheme, while when $H < 0$, then $z^{T} < z^{E}$, and participation is higher under the token-based scheme. The first term of H is positive in the low cutoff equilibrium and strictly decreasing from $(\infty, 0)$ for $P \geq 0$, while the third term is negative and decreasing in z^{T}. The LHS is then strictly decreasing in z^{E} from ∞ to a negative (potentially improper) limit, and there exists a unique z^{***} such that $H = 0$. Furthermore, by the Implicit Function theorem, $\partial H/\partial A > 0$ since δ, and consequently, the third term in H is decreasing in A, fixing z^{T}, that

$$\frac{dz^{**}}{dA} = -\frac{\partial H/\partial A}{\partial H/\partial z^{T}} > 0.$$

Since the equilibrium z^{E} in Proposition 1 is decreasing in A, while z^{***} is increasing in A, it follows that there exists a critical A^{***} such that, for $A \geq A^{***}$, $z^{E} < z^{***}$, and consequently $H > 0$. As such, when $A \geq A^{***}$, $z^{E} \leq z^{T}$ and participation is higher under the equity-based scheme. In contrast, if $A > A^{***}$, then $z^{E} > z^{***}$, and $z^{E} > z^{T}$, so that participation is higher under the token-based scheme.

Therefore, when A is sufficiently high, $A \geq \max\left\{A^{*}, A^{***}\right\}$, user participation is higher under the equity-based scheme.

We next consider owner profit. Under the equity-based scheme when there is no subversion, the owner maximizes

$$\Pi^{E} = \sup_{\delta, c}\ \delta U + c\Phi\left(-z^{E}\right),$$

where U is the total transaction surplus. In contrast, under the token-based scheme the owner optimizes

$$\Pi^{T} = \sup_{P}\ P\Phi\left(-z^{T}\right).$$

Since the equity-based scheme can always choose $\delta = 0$ and $c = P$, it follows, by revealed preference, that the owner's profit must be (weakly) higher under the equity-based scheme. This inequality is strict once we recognize that the token price is equal to the transaction surplus of the marginal user minus the participation cost, while revenue scales with the total transaction surplus through the variable fee, δ.

Finally, we consider social surplus. Social surplus under the equity-based scheme without subversion and the token-based scheme are both given by

$$U_0 = U - \kappa \Phi(-z),$$

where z is z^E under the equity-based scheme and z^T under the token-based scheme. It is straightforward to see that

$$\begin{aligned}
\frac{1}{U}\frac{dU_0}{dz} &= \frac{\kappa \Phi(-z)}{U}\frac{\phi(-z)}{\Phi(-z)} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z\right)} - \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z\right)} \\
&< \frac{\phi(-z)}{\Phi(-z)} - \frac{\phi\left(\eta_c \tau_\varepsilon^{-1/2} - z\right)}{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z\right)} - \frac{\phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z\right)}{\Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} - z\right)} \\
&< 0,
\end{aligned}$$

since $U_0 \geq 0$ implies that $U > \kappa \Phi\left(-z^S\right)$ and the hazard function of the normal distribution $\frac{\phi(-x)}{\Phi(-x)}$ is increasing in x.

Note that $U_0 \geq 0$ under the equity-based scheme or else

$$0 > U - \kappa \Phi(-z) > \delta U - \kappa \Phi(-z) = \Pi^E,$$

and owner profits would be negative, which cannot be optimal because the owner can always choose not to launch the platform.

Similarly, integrating the participation constraint for users, (32), we also have that

$$U > (\kappa + P)\,\Phi\left(-z^T\right) > \kappa \Phi\left(-z^T\right),$$

as required. Consequently, $U_0 \geq 0$ and $\frac{dU_0}{dz} < 0$.

Since $\frac{dU_0}{dz} < 0$, the platform with higher participation has a larger social surplus. It then follows that, for $A \geq \max\{A^*, A^{***}\}$, the equity-based scheme leads to a larger social surplus.

We now consider the case of subversion under the equity-based scheme and first compare user participation. Suppose now that A is sufficiently low that there is subversion, $A < A^*$. From Proposition 2, user participation is decreasing in γ. Consequently, given A, for sufficiently large γ, user participation is higher under the token-based scheme.

Similarly, with regard to owner profit, we recall from Proposition 2 that the owner's profit when there is subversion is decreasing in γ. It then follows, for sufficiently higher γ, that owner profit is also higher under the token-based scheme.

Finally, with regard to social surplus, we recall from the arguments above that social surplus is increasing in user participation when the surplus is nonnegative. From Proposition

2, social surplus is nonnegative when there is subversion, and from above it is nonnegative on the token platform. As such, for sufficiently high γ, social surplus is higher under the token-based scheme because user participation is higher than under the equity-based scheme with subversion.

B.5 Proof of Proposition 5

We recognize the stark contrast in performance of the equity-based scheme depending on whether there is subversion. Recall that when there is no subversion, from Proposition 4, revenue is higher under the equity-based scheme, $\Pi^E(A) \geq \Pi^T(A)$. In contrast, when there is subversion, for a severe enough degree of data abuse, that is, γ is sufficiently high, then $\Pi^T(A) \geq \Pi^E(A)$. From Proposition 1, subversion occurrs for $A < A_*^E$, where A_*^E is given by (28).

Consider now the prior belief of the developer over A. The difference in expected profit of the platform under both arrangements is

$$E\left[\Pi^T - \Pi^E\right] = E\left[\left(\Pi^T - \Pi^E\right)\mathbf{1}_{\{A \geq A_*^E\}}\right] + E\left[\left(\Pi^T - \Pi^E\right)\mathbf{1}_{\{A < A_*^E\}}\right],$$

from which follows that

$$E\left[\Pi^T - \Pi^E\right] = \Pr\left(A \geq A_*^E\right) E\left[\Pi^T - \Pi^E | A \geq A_*^E\right] + \Pr\left(A < A_*^E\right) E\left[\Pi^T - \Pi | A < A_*^E\right],$$

where $E\left[\Pi^T - \Pi | A \geq A_*^E\right] < 0$, since $E\left[\Pi^T - \Pi | A < A_*^E\right] > 0$ for sufficiently large γ from Proposition 4. Consequently, the first term is negative while the second is positive.

We next recognize that A_*^E, and consequently the probability that $A < A_*^E$, $\Pr\left(A < A_*^E\right)$, is increasing in γ, since the more severe the temptation is to subvert the platform, the more difficult it is to operate without abusing user data at $t = 2$. In addition, from Proposition 4, the owner's profit, conditional on subversion, is decreasing in γ. Therefore if the prior belief, $G(A)$, puts sufficient weight on low A realizations, for which $\Pr\left(A < A_*^E\right)$ is sufficiently large, then $E\left[\Pi^T\right] > E\left[\Pi^E\right]$. In contrast, if it puts sufficient weight on high A realizations, for which $\Pr\left(A < A_*^E\right)$ is sufficiently small, then $E\left[\Pi^T\right] < E\left[\Pi^E\right]$. Furthermore, the set of measures for which $E\left[\Pi^T\right] > E\left[\Pi^E\right]$ is increasing in γ.

Consequently, for two prior distributions, $G(A)$ and $\tilde{G}(A)$, if $\tilde{G} > G$ (in a first-order stochastic dominance sense), then the developer is more likely to adopt the token-based scheme under G than under $\tilde{G}$. Furthermore, the set of priors for which the developer will choose the token-based scheme is increasing in γ. In the special case of a normal prior with fixed precision τ_A, then there exists a prior mean, $\bar{A}^c$, such that the developer chooses the equity-based scheme if $\bar{A} \geq \bar{A}^c(\gamma)$ and the token-based scheme.

B.6 Proof of Proposition 6

We first examine the decision of a user to purchase the token. We first recognize that each user's expectation about P_{t+1}, $E\left[P_{t+1}\middle|\,\mathcal{I}_t\right]$, depends on each user's expectation of A_{t+1}. By the Bayes' Rule, it is straightforward to conclude that the conditional posterior of users about A_{t+1} after observing A_t and Q_t is Gaussian with

$$
\begin{aligned}
E\left[A_{t+1}\middle|\,\mathcal{I}_t\right] &= A_t + \mu + \frac{\tau_Q}{\tau_\varepsilon + \tau_Q} Q_t, \\
Var\left[A_{t+1}\middle|\,\mathcal{I}_t\right] &= \frac{1}{\tau_\varepsilon + \tau_Q}.
\end{aligned}
$$

We define τ as the stopping time, at which the platform fails as a result of the breakdown of the token market. We shall derive the conditions that determine τ later. Conditional on $t < \tau$, the expected utility of user i, who chooses to purchase the token at t, from transacting with another user is

$$
E\left[U_{i,t}\ |\mathcal{I}_t,\ \tau > t,\ A_{it},\ \text{matching with user } j\right] = e^{(1-\eta_c)A_{i,t}} E\left[e^{\eta_c A_{j,t}}\ |\mathcal{I}_t\right],
$$

which is monotonically increasing with the user's own endowment $A_{i,t}$. Note that $E\left[e^{\eta_c A_{j,t}}\ \middle|\ \mathcal{I}_t\right]$ is independent of $A_{i,t}$ but dependent on the strategies used by other users. It then follows that user i will follow a cutoff strategy that is monotonic in its own type $A_{i,t}$.

Suppose that every user uses a cutoff strategy with a threshold of $\hat{A}_t$. Then, the expected utility of user i is

$$
E\left[U_{i,t}|\mathcal{I}_t, \tau > t\right] = e^{(1-\eta_c)A_{i,t} + \eta_c A_t + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A_t - \hat{A}_t}{\tau_\varepsilon^{-1/2}}\right) \mathbf{1}_{\{\tau > t\}},
$$

since losing a transaction is independent of the identities of the two transacting parties.

To determine the equilibrium threshold, consider a user with the critical endowment $A_{it} = \hat{A}_t$. As this marginal user must be indifferent to his purchase choice, it follows that

$$
E\left[U_{i,t} + P_{t+1}\middle|\, \mathcal{I}_t, A_{it} = \hat{A}_t\right] = RP_t + \kappa,
$$

which is equivalent to

$$
e^{(1-\eta_c)A_{i,t} + \eta_c A_t + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A_t - \hat{A}_t}{\tau_\varepsilon^{-1/2}}\right) \mathbf{1}_{\{\tau > t\}} + E\left[P_{t+1}\middle|\, \mathcal{I}_t\right] = RP_t + \kappa, \tag{35}
$$

with $A_{i,t} = \hat{A}_t$. Fixing the critical value $\hat{A}_t$, the expected token price $E\left[P_{t+1}\middle|\,\mathcal{I}_t\right]$, and the price P_t, we see that the LHS of equation (35) is monotonically increasing in $A_{i,t}$, since $1 - \eta_c > 0$. This confirms the optimality of the cutoff strategy that users with $A_{i,t} \geq \hat{A}_t$

acquire the token to join the platform, and users with $A_{i,t} < \hat{A}_t$ do not. Since $A_{i,t} = A_t + \varepsilon_{i,t}$, it then follows that a fraction $\Phi\left(-\sqrt{\tau_\varepsilon}\left(\hat{A}_t - A_t\right)\right)$ of the users enter the platform, and a fraction $\Phi\left(\sqrt{\tau_\varepsilon}\left(\hat{A}_t - A_t\right)\right)$ choose not to. As one can see, it is the integral over the idiosyncratic endowment of users ε_i that determines the fraction of potential users on the platform.

By substituting P_t from equation (13) into equation (35), we obtain an equation to determine the equilibrium cutoff $\hat{A}_t = \hat{A}_t(\mathcal{I}_t)$:

$$
\begin{aligned}
& e^{A_t + (1-\eta_c)\left(\hat{A}_t - A_t\right) + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A_t - \hat{A}_t}{\tau_\varepsilon^{-1/2}}\right) \mathbf{1}_{\{\tau > t\}} + E\left[P_{t+1} \middle| \mathcal{I}_t\right] \\
= \; & e^{\frac{\sqrt{\tau_\varepsilon}}{\lambda_P}\left(A_t - \hat{A}_t\right) - \frac{1}{\lambda_P} y_t} + \kappa. \qquad (36)
\end{aligned}
$$

Define $z_t = \sqrt{\tau_\varepsilon}\left(\hat{A}_t - A_t\right)$, which determines the population that buys the token. We can rewrite equation (36) as

$$
\begin{aligned}
& e^{\left[(1-\eta_c)\tau_\varepsilon^{-1/2} + \frac{1}{\lambda_P}\right] z_t + A_t + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} - z_t\right) \mathbf{1}_{\{\tau > t\}} \qquad (37) \\
& \quad + e^{\frac{1}{\lambda_P} z_t}\left(E\left[P_{t+1} \middle| \mathcal{I}_t\right] - \kappa\right) \; = \; e^{-\frac{1}{\lambda_P} y_t}.
\end{aligned}
$$

Note that the first term in the LHS of equation (37) has a humped shape with respect to z_t, and the second term is an exponential function of z_t with a coefficient that may be either positive or negative. As the RHS of equation (37) is constant with respect to z_t, this equation may have zero, one, two, or three roots:

- If $E\left[P_{t+1} \middle| \mathcal{I}_t\right] - \kappa \leq 0$, the LHS has a humped shape with a maximum at $\bar{z}$, and it may intersect with the RHS at zero or two points:

  1. If $LHS(\bar{z}) < RHS$, then equation (37) has no root.
  2. If $LHS(\bar{z}) > RHS$, then equation (37) has two roots.

- If $E\left[P_{t+1} \middle| \mathcal{I}_t\right] - \kappa > 0$, the LHS is nonmonotonic with $LHS(-\infty) = 0$, $LHS(\infty) = \infty$, and one local maxium $\ddot{z}$ and one local minimum $\dot{z}$ in $(-\infty, \infty)$, and it may intersect the RHS at one or three points:

  3. If $RHS < LHS(\dot{z})$ or if $RHS > LHS(\ddot{z})$, then equation (37) has one root.
  4. If $LHS(\dot{z}) < RHS < LHS(\ddot{z})$, then equation (37) has three roots.

In the first scenario outlined above, there is no equilibrium, and the token market breaks down. Note that A_t shifts up and down the left-hand side of equation (37). Thus, equation

(37) has no root when A_t is sufficiently small. For this situation to occur, the speculative motive, $E[P_{t+1} \mid \mathcal{I}_t] - \kappa$, must be nonpositive; otherwise equation (37) has one or three roots. This condition is also satisfied when A_t is sufficiently small because $E[P_{t+1} \mid \mathcal{I}_t]$ is increasing with A_t. Thus, the token market breaks down when A_t falls below a certain critical level, which we denote as $A_*(y_t, Q_t)$. Thus, the stopping time τ of the platform's disbandment is

$$\tau = \{\inf \ t : \ A_t < A_*(y_t, Q_t)\}.$$

We next consider how user optimism Q_t impacts the market breakdown region. Since user optimism Q_t raises each user's estimate of the resale value of the token at date $t+1$, it raises user participation and the token price at date t. Since Q_t is i.i.d., this is the only impact of an increase in user optimism. As such, it shifts down the market breakdown threshold, $A_*(y_t, Q_t,)$, for any given pair of y_t, and facilitates breakdown when $E[P_{t+1} \mid \mathcal{I}_t] - \kappa$ is nonpositive.

Similarly, an increase in the user participation cost, κ, deters user participation at all dates and therefore exacerbates the market breakdown by both increasing the cost today and lowering the expected retrade value of the token tomorrow through reduced participation in the future. As such, it also shifts up $A_*(y_t, Q_t)$.

Note that because the only difference among users is the value of their transaction benefit, $E[U_{i,t} \mid \mathcal{I}_t, \tau > t]$, which is monotonically increasing in $A_{i,t}$ regardless of the mass of users that join the platform, it follows that, regardless of the strategies of other users, it is always optimal for each user i to follow a cutoff strategy.

Finally, since user optimism Q_t enters into the user's problem by raising the expected resale token price, it raises user participation and the token price. Further, because a stronger demand fundamental A_t shifts out the hump-shaped LHS in (36), it follows that it drives down cutoff $\hat{A}_t$ in the lowest cutoff equilibrium, $\frac{d\hat{A}_t}{dA_t} < 0$; as such, user participation and consequently the token price, are also increasing in A_t.

B.7 Proof of Proposition 7

The first-order conditions of user i's optimization problem in (16) with respect to $C_i(i)$ and $C_j(i)$ at an interior point are:

$$C_i(i) \ : \ \frac{1-\eta_c}{C_i(i)} U(C_i(i), C_j(i)) = \theta_i p_i, \tag{38}$$

$$C_j(i) \ : \ \frac{\eta_c}{C_j(i)} U(C_i(i), C_j(i)) = \theta_i p_j, \tag{39}$$

where θ_i is the Lagrange multiplier for the budget constraint. Rewriting (39) as

$$\eta_c U(C_i(i), C_j(i)) \ = \ \theta_i p_j C_j(i).$$

Dividing equation (38) by this expression leads to $\frac{\eta_c}{1-\eta_c} = \frac{p_j C_j(i)}{p_i C_i(i)}$, which in a symmetric equilibrium implies $p_j C_j(i) = \frac{\eta_c}{1-\eta_c} p_i C_i(i)$. By substituting this equation back to the user's budget constraint in (16), we obtain:

$$C_i(i) = (1-\eta_c) e^{A_i}.$$

The market-clearing for the user's good requires that $C_i(i) + C_i(j) = e^{A_i}$, which implies that $C_i(j) = \eta_c e^{A_i}$.

The first-order condition in equation (38) also gives the price of the good produced by user i. Since the user's budget constraint in (16) is entirely in nominal terms, the price system is only identified up to θ_i, the Lagrange multiplier. We therefore normalize θ_i to 1. It follows that:

$$p_i = \frac{1-\eta_c}{C_i(i)} U(C_i(i), C_j(i)) = e^{\eta_c(A_j - A_i)}. \tag{40}$$

Furthermore, given equation (1), it follows that since $C_i(i) = (1-\eta_c) e^{A_i}$ and $C_j(i) = \eta_c e^{A_j}$:

$$U(C_i(i), C_j(i)) = e^{(1-\eta_c)A_i} e^{\eta_c A_j} = p_i e^{A_i},$$

from substituting with the user's budget constraint at $t = 2$.

It then follows that, conditional on matching with another user on the platform, the expected utility of user i conditional on his endowment A_i and a successful match is:

$$E[U(C_i(i), C_j(i)) | A_i, \text{matching}] = e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \frac{\Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right)},$$

and, since the probability of meeting another holder of the token is $\Phi\left(\frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right)$, the expected utility of user i is:

$$E[U(C_i(i), C_j(i)) | A_i, A] = e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right).$$

Finally, the ex ante expected utility benefit of a user before it learns its endowment A_i is

$$\begin{aligned}
U_0 &= E[E[U_i | A_i, A] | A] \\
&= E\left[e^{(1-\eta_c)A_i + \eta_c A + \frac{1}{2}\eta_c^2 \tau_\varepsilon^{-1}} \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right) \mid A\right] \\
&= e^{A + \frac{1}{2}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}} \Phi\left((1-\eta_c)\tau_\varepsilon^{-1/2} + \frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right) \Phi\left(\eta_c \tau_\varepsilon^{-1/2} + \frac{A-\hat{A}}{\tau_\varepsilon^{-1/2}}\right).
\end{aligned}$$